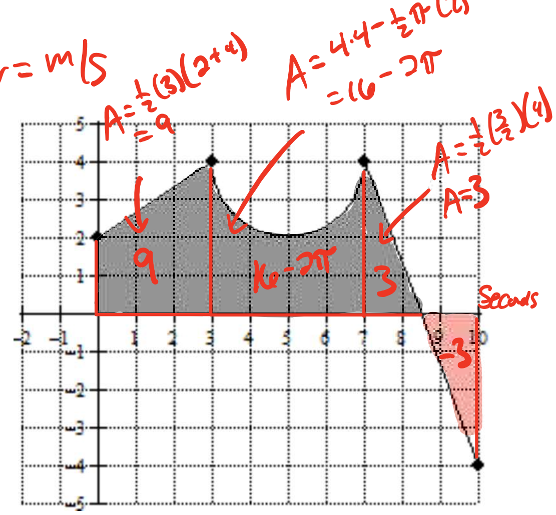
Name____Date____________________Class__________

Homework 6.5

The graph to the right represents the velocity, $v(t)$ in meters per second, of a particle that is moving along the x – axis on the time interval $0 \le t \le 10$. The initial position of the particle at time $t = 0$ is 12.

$v = m/s$

$p(0) = 12$



1. On what interval(s) of time is the particle moving to the left and to the right? Justify your answer.

The particle is moving right on $[0, 8.5)$ b/c $v(t) > 0$.

The particle is moving left on $(8.5, 10]$ b/c $v(t) < 0$.

2. What is the total distance that the particle has traveled on the time interval $0 \le t \le 7$. Leave your answer in terms of π. Indicate units of measure.

$$\text{Total Distance} = \int_0^7 |v(t)|\,dt = \int_0^3 |v(t)|\,dt + \int_3^7 |v(t)|\,dt$$
$$= 9 + 16 - 2\pi$$
$$= (25 - 2\pi) \text{ meters}$$

3. What is the net distance that the particle travels on the interval $5 \le t \le 10$? Round your answer to the nearest thousandth. Indicate units of measure.

$$\text{Net Distance} = \int_5^{10} v(t)\,dt = \int_5^7 v(t)\,dt + \int_7^{8.5} v(t)\,dt + \int_{8.5}^{10} v(t)\,dt$$
$$= \frac{1}{2}(16 - 2\pi) + (3) + (-3)$$
$$= (8 - \pi) \text{ meters}$$

4. What is the acceleration of the particle at time $t = 2$? Indicate units of measure.

$a(2) = v'(2) = 2/3 \text{ meters/second}^2$ (slope of curve v)

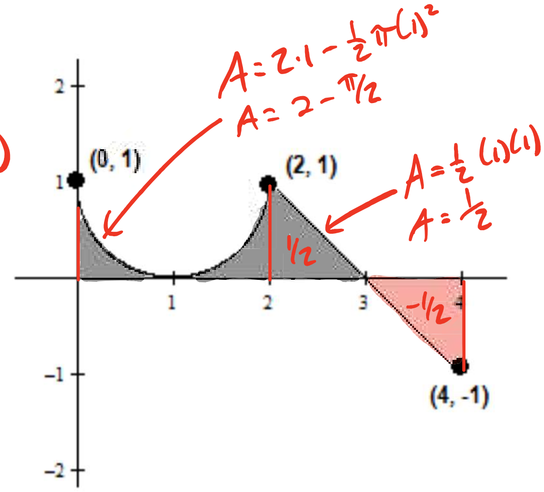
5. What is the position of the particle at time $t = 5$? Indicate units of measure.

FIND $P(5)$

$$\int_0^5 v(t)\,dt = p(5) - p(0)$$
$$\int_0^3 v(t)\,dt + \int_3^5 v(t)\,dt = p(5) - 12$$
$$9 + \frac{1}{2}(16 - 2\pi) = p(5) - 12$$
$$9 + 8 - \pi + 12 = p(5)$$
$$29 - \pi = p(5)$$

$\boxed{p(5) = (29 - \pi) \text{ meters}}$

Pictured to the right is the graph of a function which represents a particle's velocity on the interval [0, 4]. Answer the following questions.



6. For what values is the particle moving to the right? Justify your answer.

The particle is moving to the right on $(0,1) \cup (1,3)$ because $v(t) > 0$.

7. For what values is the particle moving to the left? Justify your answer.

The particle is moving to the left on $(3,4)$ because $v(t) < 0$.

8. For what values is the speed of the particle increasing? Justify your answer.

The speed is increasing on $(1,2)$ and $(3,4)$ because $v(t)$ and $a(t)$ have the same signs.

9. For what values is the speed of the particle decreasing? Justify your answer.

The speed is decreasing on $(0,1)$ and $(2,3)$ because $v(t)$ and $a(t)$ have different signs.

10. What is the net distance that the particle travels on the interval [0, 4]?

$$\text{Net Distance} = \int_0^4 v(t)\,dt = \int_0^2 v(t)\,dt + \int_2^3 v(t)\,dt + \int_3^4 v(t)\,dt$$
$$= \left(2 - \frac{\pi}{2}\right) + \left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right)$$
$$= \left(2 - \frac{\pi}{2}\right) \text{ meters}$$

11. What is the total distance that the particle travels on the interval [0, 4]?

$$\text{Total Distance} = \int_0^4 |v(t)|\,dt = \int_0^2 |v(t)|\,dt + \int_2^3 |v(t)|\,dt + \int_3^4 |v(t)|\,dt$$
$$= \left|2 - \frac{\pi}{2}\right| + \left|\frac{1}{2}\right| + \left|-\frac{1}{2}\right|$$
$$= \left(3 - \frac{\pi}{2}\right) \text{ meters}$$

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity, v, measured in feet per second, and acceleration, a, measured in feet per second per second, are continuous and differentiable functions on $0 \le t \le 60$. The table below shows selected values of these functions.

25 10 25

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

12. Using appropriate units, explain the meaning of $\int_0^{60} |v(t)|\,dt$ in terms of the car's motion. Approximate this integral using a midpoint approximation with three subintervals as determined by the table.

$\int_0^{60} |v(t)|\,dt \approx |25(-30)| + |10(-14)| + |25(0)| \approx 750 + 140 + 0 \approx 890$ feet

$\int_0^{60} |v(t)|\,dt$ is the total distance the car travels from $t=0$ seconds to $t=60$ seconds

13. Using appropriate units, explain the meaning of $\int_{15}^{50} a(t)\,dt$ in terms of the car's motion. Find the exact value of the integral.

$\int_{15}^{50} a(t)\,dt = v(50) - v(15) = 0 - (-30) = 30$ feet/sec

$\int_{15}^{50} a(t)\,dt$ is the change in velocity from $t=15$ seconds to $t=50$ seconds.

14. Is there a value of t such that $a'(t) = 0$? If so, identify an interval on which such a value of t exists? Justify your reasoning.

Rolle's Theorem guarantees a value of c on $(25,35)$ such that $a'(t)=0$

b/c (a) $a(t)$ is continuous on $[25,35]$

(b) $a(t)$ is differentiable on $(25,35)$

(c) $a(25) = 2 = a(35)$

15. Using appropriate units, approximate the value of $v'(31)$. What does this value say about the motion of the car at $t = 31$.

$v'(31) \approx \dfrac{v(30) - v(35)}{30 - 35} = \dfrac{(-14)-(-10)}{-5} = \dfrac{-4}{-5} = \dfrac{4}{5}$ feet/second^2

Since $v'(31) > 0$, the velocity is increasing at $t = 31$ seconds.

16. Using appropriate units, find the value and explain the meaning of $\frac{1}{35}\int_{25}^{60} a(t)\,dt$.

$\frac{1}{35}\int_{25}^{60} a(t)\,dt = \frac{1}{35}[v(60) - v(25)]$

$= \frac{1}{35}[10 - (-20)]$

$= \frac{1}{35}(30)$

$= \frac{6}{7}$ feet/second

$\frac{1}{35}\int_{25}^{60} a(t)\,dt$ is the average velocity of the car from $t=25$ seconds to $t=60$ seconds.