Name____ **Date**_______________________ **Class**____________

Homework 7.1

Find the derivative of each of the following functions defined by integrals.

1. $g(x) = \int_{2}^{3x}(2t+3)dt$

$g'(x) = [2(3x)+3]\cdot(3x)'$

$= 3(6x+3)$

$g'(x) = 18x+9$

2. $h(x) = \int_{-2}^{x^4} 3\sqrt{t}dt$

$h'(x) = 3\sqrt{x^4}\cdot(x^4)'$

$= 4x^3\cdot 3\cdot x^2$

$h'(x) = 12x^5$

3. $f(x) = \int_{2x}^{-1}(t^2+2t)dt$

$f'(x) = -\int_{-1}^{2x}(t^2+2t)dt$

$= -((2x)^2+2(2x))\cdot 2$

$= -(4x^2+4x)\cdot 2$

$f'(x) = -8x^2-8x$

4. $H(x) = \int_{-5}^{\cos x} 2t^2 dt$

$H'(x) = 2\cos^2 x\cdot(-\sin x)$

$H'(x) = -2\sin x\cos^2 x$

5. $P(x) = \int_{2}^{x^2+2x}(3t-2)dt$

$P'(x) = (3(x^2+2x)-2)\cdot(2x+2)$

$P'(x) = (3x^2+6x-2)(2x+2)$

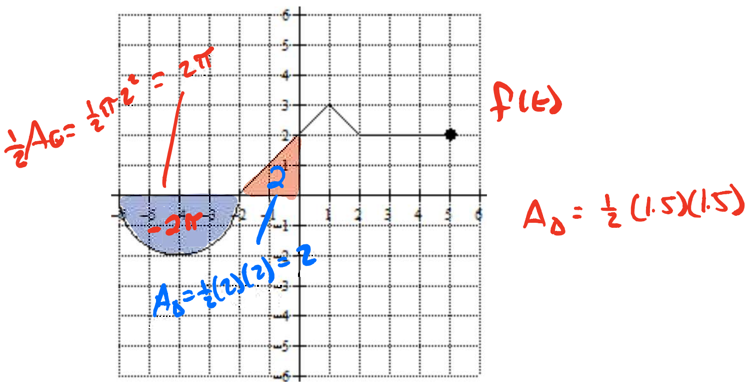
6. $f(x) = \int_{\ln x}^{2}(e^t+t)dt$

$f(x) = -\int_{2}^{\ln x}(e^t+t)dt$

$= -(e^{\ln x}+\ln x)\cdot\frac{1}{x}$

$f(x) = -\frac{x+\ln x}{x}$

Pictured is the graph of $f(t)$ and $F(x) = \int_{-6}^{2x} f(t)\,dt$. Use the graph and $F(x)$ to answer the questions 7 – 11.



7. Find the value of $F(0)$.

$F(0) = \int_{-6}^{2\cdot 0} f(t)dt$

$= -2\pi + 2$

8. Find the value of $F\left(-\frac{1}{2}\right)$.

$F\left(-\frac{1}{2}\right) = \int_{-6}^{2(-\frac{1}{2})} f(t)dt = \int_{-6}^{-1} f(t)dt$

$= -\frac{1}{2}\pi(2)^2 + \frac{1}{2}(1)(1)$

$= -2\pi + \frac{1}{2}$

9. Find the value of $F'(-2)$.

$F'(x) = f(2x)\cdot(2x)'$

$F'(x) = 2f(2x)$

$F'(-2) = 2f(2\cdot(-2))$

$= 2f(-4)$

$= 2\cdot(-2)$

$= -4$

10. Find the value of $F'(2.5)$.

$F'(2.5) = 2f(2\cdot 2.5)$

$= 2f(5)$

$= 2\cdot 2$

$= 4$

11. Find the value of $F''(0)$

$F''(x) = 2f'(2x)\cdot(2x)'$

$F''(x) = 4f'(2x)$

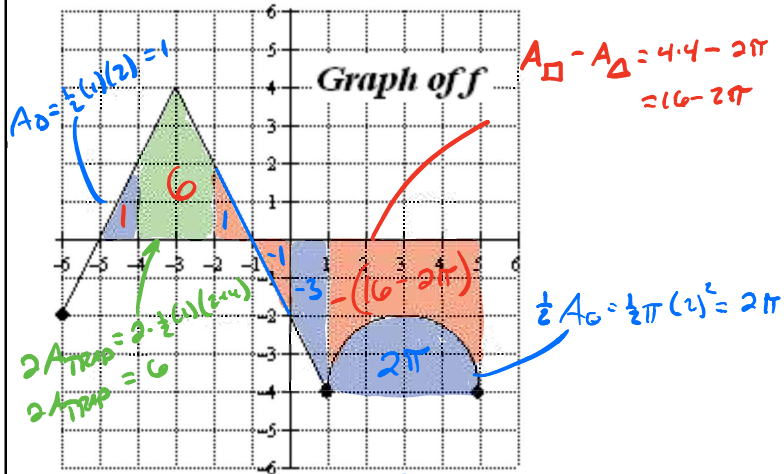
$F''(0) = 4f'(2\cdot 0)$

$= 4\cdot f'(0)$

$= 4\cdot 1$

$F''(0) = 4$

Pictured is the graph of f and $G(x) = \int_{-2}^{x} f(t)\,dt$. Use the graph to answer questions 12 – 15.



12. Find the value of $G(3)$. $= \int_{-2}^{3} f(t)dt$

$= 1 + (-1) + (-3) + -\frac{1}{2}(16 - 2\pi)$

$= -3 - 8 + \pi$

$= \pi - 11$

13. Find the value of $G(-4)$. $= \int_{-2}^{-4} f(t)dt$

$= -\int_{-4}^{-2} f(t)dt$

$= -[6]$

$= -6$

14. Find the value of $G'(-2)$. $\int_{-2}^{x} f(t)dt$

$G'(x) = f(x)\cdot x'$

$G'(x) = f(x)$

$G'(-2) = f(-2) = 2$

15. Find the value of $G''(-5)$.

$G'(x) = f(x)$

$G''(x) = f'(x)$

$G''(-5) = f'(-5) = 2$

(slope of f at x=-5)

If $g(x) = \int_0^x t^3 e^t dt$, find each of the following values in questions 16 – 17.

16. Find the value of $g'(1)$.

$g'(x) = x^3 e^x \cdot x'$

$g'(x) = x^3 e^x$

$g'(1) = (1)^3 e^1 = e$

17. Find the value of $g''(1)$.

$g'(x) = \underbrace{x^3}\underbrace{e^x}$

$g''(x) = 3x^2 e^x + x^3 e^x$

$g''(1) = 3(1)^2 e^{(1)} + (1)^3 e^{(1)}$

$= 3e + e$

$g''(1) = 4e$

If $h(x) = \int_{x^2}^2 \sqrt{1+t^4}\,dt$, find each of the following values in questions 18 – 19.

18. Find $h'(x)$.

$h(x) = -\int_2^{x^2} \sqrt{1+t^4}\,dt$

$h'(x) = -\sqrt{1+(x^2)^4} \cdot (x^2)'$

$h'(x) = -2x\sqrt{1+x^8}$

19. Find $h''(1)$.

$h'(x) = \underbrace{-2x}\underbrace{(1+x^8)^{1/2}}$

$h''(x) = -2(1+x^8)^{1/2} - 2x \cdot \overbrace{\tfrac{1}{2}(1+x^8)^{-1/2} \cdot 8x^7}^{\text{chain}}$

$h''(x) = -2\sqrt{1+x^8} - \dfrac{8x^8}{\sqrt{1+x^8}}$

$h''(1) = -2\sqrt{1+(1)^8} - \dfrac{8(1)^8}{\sqrt{1+(1)^8}}$

$= \dfrac{-2\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}} - \dfrac{8}{\sqrt{2}}$

$= \dfrac{-4-8}{\sqrt{2}}$

$h''(1) = \dfrac{-12}{\sqrt{2}}$