

Part II: Integrate each definite integral with u-substitution.

#1)  $\int_0^2 (x^2 - 1)^5 2x \, dx$

$$\begin{aligned} u &= x^2 - 1 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

$$\begin{aligned} &= \int_{-1}^3 u^5 \cancel{2x} \left( \frac{du}{2x} \right) \\ &= \int_{-1}^3 u^5 \, du \\ &= \left. \frac{1}{6} u^6 \right|_{-1}^3 \\ &= \left[ \frac{1}{6} (3)^6 \right] - \left[ \frac{1}{6} (-1)^6 \right] \\ &= \left[ \frac{1}{6} (729) \right] - \left[ \frac{1}{6} (1) \right] \\ &= \frac{729}{6} - \frac{1}{6} \\ &= \frac{728}{6} \\ &= \frac{364}{3} \end{aligned}$$

#2)  $\int_0^1 e^{x^2} 2x \, dx$

$$\begin{aligned} u &= x^2 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

$$\begin{aligned} &= \int_0^1 e^u \cancel{2x} \left( \frac{du}{2x} \right) \\ &= \int_0^1 e^u \, du \\ &= e^u \Big|_0^1 \\ &= [e^1] - [e^0] \\ &= e - 1 \end{aligned}$$

#3)  $\int_{-3}^3 \frac{2x}{x^2-1} \, dx$

$$\begin{aligned} u &= x^2 - 1 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

$$\begin{aligned} &= \int_{-3}^3 \frac{\cancel{2x}}{u} \left( \frac{du}{\cancel{2x}} \right) \\ &= \int_{-3}^3 \frac{1}{u} \, du \\ &= \text{sad face} \end{aligned}$$

#4)  $\int_{-1}^1 (x^2 - 1)^5 x \, dx$

$$\begin{aligned} u &= x^2 - 1 \\ \frac{du}{dx} &= 2x \\ du &= 2x \, dx \\ \frac{du}{2x} &= dx \end{aligned}$$

$$\begin{aligned} &= \int_{-1}^1 u^5 \cancel{x} \left( \frac{du}{2x} \right) \\ &= \frac{1}{2} \int_{-1}^1 u^5 \, du \\ &= \text{sad face} \end{aligned}$$

$$\#5) \int_1^8 e^{x^3} 3x^2 dx = \int_1^8 e^u 3x^2 \left(\frac{du}{3x^2}\right)$$

$$\begin{aligned} u &= x^3 \\ \frac{du}{dx} &= 3x^2 \\ du &= 3x^2 dx \\ \frac{du}{3x^2} &= dx \end{aligned}$$

$$\begin{aligned} &= \int_1^8 e^u du \\ &= e^u \Big|_1^8 \\ &= e^8 - e^1 \\ &= e^8 - e \end{aligned}$$

$$\#6) \int_{-2}^2 \frac{x^2}{x^3+2} dx = \int_{-6}^{10} \frac{1}{u} \left(\frac{du}{3x^2}\right)$$

$$\begin{aligned} u &= x^3+2 \\ \frac{du}{dx} &= 3x^2 \\ du &= 3x^2 dx \\ \frac{du}{3x^2} &= dx \end{aligned}$$

$$\begin{aligned} &= \frac{1}{3} \int_{-6}^{10} \frac{1}{u} du \\ &= \frac{1}{3} \ln|u| \Big|_{-6}^{10} \\ &= \frac{1}{3} \ln|u| \Big|_{-6}^{10} \\ &= \left[ \frac{1}{3} \ln|10| \right] - \left[ \frac{1}{3} \ln|-6| \right] \\ &= \frac{1}{3} [\ln 10 - \ln 6] \\ &= \frac{1}{3} \ln\left(\frac{5}{3}\right) \end{aligned}$$

### Bathroom Tissue

#7) Adding to his line of products for *The Slightly Used Company*, George starts selling bathroom tissue. *Slightly Used's* marginal cost function is  $MC(x) = \frac{1}{4x+2}$  and its fixed costs are \$4. Find the cost function.

$$(0.4)$$

$$\begin{aligned} C(x) &= \int MC(x) dx \\ &= \int \frac{1}{4x+2} dx \\ &= \int \frac{1}{u} \left(\frac{du}{4}\right) \\ &= \frac{1}{4} \int \frac{1}{u} du \\ &= \frac{1}{4} \ln|u| + C \\ C(x) &= \frac{1}{4} \ln|4x+2| + C \end{aligned}$$

$$\begin{aligned} u &= 4x+2 \\ \frac{du}{dx} &= 4 \\ du &= 4 dx \\ \frac{du}{4} &= dx \end{aligned}$$

$$\begin{aligned} 4 &= \frac{1}{4} \ln|4(0)+2| + C \\ 4 &= \frac{1}{4} \ln|2| + C \\ 16 &= \ln 2 + C \\ 16 - \ln 2 &= C \end{aligned}$$

$$C(x) = \frac{1}{4} \ln|4x+2| + 16 - \ln 2$$

### Pluckable Hairs

#8) The number of pluckable hairs on George's ears is expected to be  $P(x) = x(x^2+4)^{-1/2}$  hairs after  $x$  months. Find the average number of pluckable hairs between month  $x=0$  and month  $x=8$ .

$$\begin{aligned} \text{Average Pluckable Hairs} &= \frac{1}{8-0} \int_0^8 x(x^2+4)^{-1/2} dx \\ &= \frac{1}{8} \int_0^8 x u^{-1/2} \left(\frac{du}{2x}\right) \\ &= \frac{1}{8} \cdot \frac{1}{2} \int_4^{68} u^{-1/2} du \\ &= \frac{1}{8} u^{1/2} \Big|_4^{68} \\ &= \left[ \frac{1}{8} \sqrt{68} \right] - \left[ \frac{1}{8} \sqrt{4} \right] \\ &= \frac{1}{8} \sqrt{68} - \frac{1}{4} \end{aligned}$$

$$\begin{aligned} u &= x^2+4 \\ \frac{du}{dx} &= 2x \\ du &= 2x dx \\ \frac{du}{2x} &= dx \end{aligned}$$

$$\text{Average Pluckable Hairs} \approx .781 \text{ per month}$$

The average number of pluckable hairs from month 0 to 8 is .781 per month.

### Alliteration – The Prequel

#9) George sells sails for snail sized sailboats. His sales of sails for snail sized sailboats during week  $x$  are given by  $S(x) = \frac{1}{x+4}$  in hundreds. Find the average sales of sails for snail sized sailboats from week  $x = 1$  to week  $x = 4$ . (Don't forget your answer is in hundreds, noob.)

$$\begin{aligned} AS &= \frac{1}{4 \cdot 1} \int_1^4 \frac{1}{x+4} dx \\ &= \frac{1}{3} \int_5^8 \frac{1}{u} du \\ &= \frac{1}{3} \ln|u| \Big|_5^8 \\ &= \left[ \frac{1}{3} \ln 8 \right] - \left[ \frac{1}{3} \ln 5 \right] \\ &= \frac{1}{3} [\ln 8 - \ln 5] \\ &= \frac{1}{3} \ln \frac{8}{5} \text{ hundred} \\ &= .157 \text{ hundred} \\ &= 15.7 \end{aligned}$$

$$\begin{aligned} u &= x+4 \\ \frac{du}{dx} &= 1 \\ du &= dx \end{aligned}$$

The average sales of sails was 15.7 from week 1 to 4.

### Alliteration

#10) An experimental therapy lowers a patient's patience for patterns at the rate of  $t\sqrt{36-t^2}$  units per day, where  $t$  is the number of days since the therapy was administered (for the first six days). Find the total change in a patient's patience for patterns during the first 3 days.

$$\begin{aligned} PPP &= \int_0^3 t\sqrt{36-t^2} dt \\ &= \int_{36}^{27} \sqrt{u} \left( \frac{du}{-2t} \right) \\ &= -\frac{1}{2} \int_{36}^{27} u^{\frac{1}{2}} du \\ &= \frac{1}{2} \int_{27}^{36} u^{\frac{1}{2}} du \\ &= \frac{1}{2} \left( \frac{2}{3} \right) u^{\frac{3}{2}} \Big|_{27}^{36} \\ &= \left[ \frac{1}{3} (\sqrt{36})^3 \right] - \left[ \frac{1}{3} (\sqrt{27})^3 \right] \\ &= \left[ \frac{1}{3} (6^3) \right] - \left[ \frac{1}{3} (\sqrt{27})^3 \right] \\ &= \frac{216}{3} - \frac{(\sqrt{27})^3}{3} \\ &= \frac{2(6 - \sqrt{27})^3}{3} \\ &\approx 25.235 \text{ (CALC)} \end{aligned}$$

A patient's patience for patterns lowers by about 25 units during the first 3 days.

### Condiments

#11) George has developed a new business model for making money in the restaurant business – give away the food for free, but charge for the condiments. He is selling condiments at the rate of  $100e^{-x}$  per week after  $x$  weeks. How many condiments will be sold during the first 8 weeks?

$$\begin{aligned} C &= \int_0^8 100e^{-x} dx \\ &= \int_{-8}^0 100e^u (-du) \\ &= 100 \int_{-8}^0 e^u du \\ &= 100e^u \Big|_{-8}^0 \\ &= [100e^0] - [100e^{-8}] \\ &= 100 - \frac{100}{e^8} \end{aligned}$$

$$\begin{aligned} u &= -x \\ \frac{du}{dx} &= -1 \\ du &= -dx \\ -du &= dx \end{aligned}$$

$$C \approx 99.966 \text{ condiments sold } \text{CALC}$$

George will sell about 99.966 condiments during the first week.

### Discharging Pits

#12) George's armpits are discharging pollution into the air at the rate of  $r(t)$  liters per year given by  $r(t) = \frac{1}{t+1}$  where  $t$  is the number of years since George washed. Find the total amount of pollution discharged during the first 3 years of not washing.

$$\begin{aligned} \text{Total Pollution} &= \int_0^3 \frac{1}{t+1} dt \\ &= \int_1^4 \frac{1}{u} du \\ &= \ln|u| \Big|_1^4 \\ &= [\ln(4)] - [\ln(1)] \\ &= \ln 4 - 0 \\ &= \ln 4 \\ &\approx 1.386 \text{ liters } \text{CALC} \end{aligned}$$

$$\begin{aligned} u &= t+1 \\ \frac{du}{dt} &= 1 \\ du &= dt \end{aligned}$$

George's armpits have sent 1.4 liters of pollution into the air during the first 3 years.