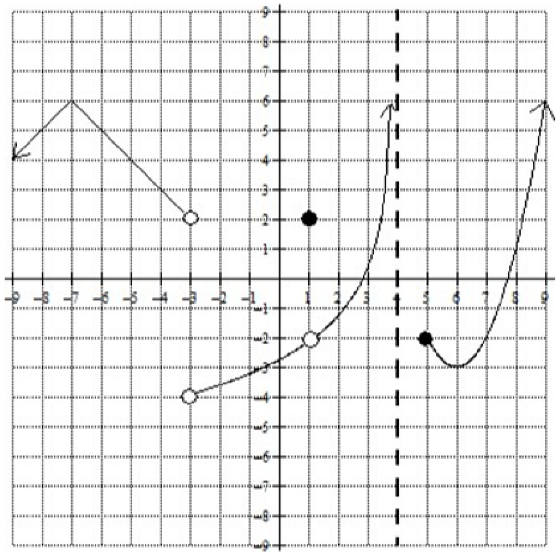
AP Calculus
Extra Practice on Limits and Multiple Choice Practice

For questions 1 – 5, refer to the graph of $f(x)$ to the right. Find the value of each indicated limit. If a limit does not exist, give a reason.

1.	$-4 + 3(-2)$ $\lim_{x\to -3^+} f(x) + \lim_{x\to 5^+} 3f(x)$	-10
2.	$\frac{1}{2}(-2) + -1$ $\lim_{x\to 1}\left[\frac{1}{2} f(x) + \cos(\pi x)\right]$	-2
3.	$\lim_{x\to 4^-} f(x) = \infty$	DNE ∞ is not number
4.	$\lim_{x\to -\infty} f(x) = -\infty$	DNE $-\infty$ is not a number
5.	$\lim_{x\to -3} f(x)$	DNE $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x)$



For questions 6 – 11, find the value of each limit analytically. If a limit does not exist, state why.

6. $\lim_{x\to 0} \frac{x^3 - 2x^2 + 3x}{x} = \lim_{x\to 0} \frac{\cancel{x}(x^2 - 2x + 3)}{\cancel{x}}$

$= (0)^2 - 2(0) + 3$

$= 3$

7. $\lim_{x\to 0} \frac{3\tan x}{x \sec x} = 3\lim_{x\to 0} \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{x}$

$= 3 \cdot 1$

$= 3$

8. $\lim_{x\to 3^+} \frac{x^2 - 4}{x^2 - x - 6} = \lim_{x\to 3^+} \frac{\cancel{(x+2)}(x-2)}{(x-3)\cancel{(x+2)}} = \infty$, DNE

x	$\frac{x-2}{x-3}$
3.1	$\frac{+}{+}$

(∞ is not a number)

9. $\lim_{x\to 2^-} \ln(-x+2) = \ln(-2+2) = \ln 0$

$= $ DNE

Argument > 0

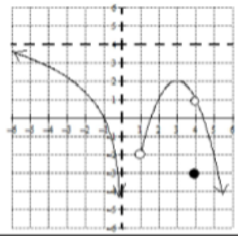
10. $\lim_{x\to \infty} \frac{x^3 - 2x^2 + 3x}{x^2 - 3x^3} = -\frac{1}{3}$

11. $\lim_{x\to \infty}\left(5 + \frac{5}{x}\right) = 5 + 0 = 5$

For question 12 – 16, use the equation $g(x)$ below and the graph of the function $f(x)$.

$$g(x)=\begin{cases} 3|x+3|, & x<-2 \\ \cos\left(\frac{\pi x}{2}\right), & -2\le x<2 \\ ax^2+2x, & x\ge 2 \end{cases}$$

Graph of $f(x)$



12. Is $g(x)$ continuous at $x = -2$. [Base your response on the three part definition of continuity.]

I. $g(-2) = \cos(-\pi) = -1$ $\therefore$ $g(-2)$ is defined

II $\lim\limits_{x\to -2^-} 3|x+3| = 3|-2+3| = 3|1| = 3$

$\lim\limits_{x\to -2^+} \cos\left(\frac{\pi x}{2}\right) = \cos\left(\frac{\pi(-2)}{2}\right) = -1$

$\therefore$ $\lim\limits_{x\to -2} g(x)$ dne $\quad$ $\therefore$ $g(x)$ is not continuous at $x = -2$

13. For what value(s) of a is $g(x)$ continuous at $x = 2$?

$\lim\limits_{x\to 2^-} \cos\left(\frac{\pi x}{2}\right) = \lim\limits_{x\to 2^+} (ax^2+2x)$

$\cos\left(\frac{\pi \cdot 2}{2}\right) = a(2)^2+2(2)$

$-1 = 4a+4$

$-5 = 4a$ $\rightarrow$ $a = -5/4$

14. For what value(s) of b is the function $f(x)$ discontinuous? At which of these values does $\lim\limits_{x\to b} f(x)$ exist? Explain your reasoning.

$f(0)$ and $f(1)$ are undefined.

$\lim\limits_{x\to 4} f(x) = 1 \neq f(4) = -3$

$\therefore$ $f(x)$ is discontinuous at $x=0$, $x=1$, and $x=4$

15. Find $\lim\limits_{x\to 2^+}[g(x)+2f(x)]$. $= \lim\limits_{x\to 2^+} g(x) + 2\lim\limits_{x\to 2^+} f(x)$

$= a(2)^2+2(2) + 2\cdot 1$

$= 4a+4+2$

$= 4a+6$

16. Which of the following limits do(es) not exist? Give a reason for your answers.

$\lim\limits_{x\to 1} f(x)$	$\lim\limits_{x\to 4} f(x) = 1$	$\lim\limits_{x\to 0^-} f(x) = -\infty$
DNE $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$	Exists	DNE $-\infty$ is not a number

17. Find the values of k and m so that the function below is continuous on the interval $(-\infty,\infty)$.

$$f(x)=\begin{cases} x^2-kx+3, & x<-2 \\ 2x-3, & -2\le x\le 3 \\ 3-2m, & x>3 \end{cases}$$

$\lim\limits_{x\to -2^-}(x^2-kx+3)=\lim\limits_{x\to -2^+}(2x-3)$

$(-2)^2-k(-2)+3=2(-2)-3$

$4+2k+3=-4-3$

$2k+7=-7$

$2k=-14$

$k=-7$

$\lim\limits_{x\to 3^-}(2x-3)=\lim\limits_{x\to 3^+}(3-2m)$

$2(3)-3=3-2m$

$6-3=3-2m$

$3=3-2m$

$0=-2m$

$0=m$

18. $\lim\limits_{x\to 0}\dfrac{4x-3}{7x+1}=$

A. ∞ B. $-\infty$ C. 0 D. $\frac{4}{7}$ E. -3

19. $\lim\limits_{x\to \frac{1}{3}}\dfrac{9x^2-1}{3x-1}=$

A. ∞ B. $-\infty$ C. 0 D. 2 E. 3

20. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}=$

A. 4 B. 0 C. 1 D. 3 E. 2

21. The function $G(x)=\begin{cases} x-3, & x>2 \\ -5, & x=2 \\ 3x-7, & x<2 \end{cases}$ is not continuous at $x=2$ because…

A. $G(2)$ is not defined

B. $\lim\limits_{x\to 2}G(x)$ does not exist

C. $\lim\limits_{x\to 2}G(x)\neq G(2)$

D. Only reasons B and C

E. All of the above reasons.

22. $\lim\limits_{x\to \infty}\dfrac{-3x^2+7x^3+2}{2x^3-3x^2+5}=$

A. ∞ B. $-\infty$ C. 1 D. $\frac{7}{2}$ E. $-\frac{3}{2}$