Free Response Practice #1 Calculator Permitted

Consider the function $h(x) = \frac{-2x-\sin x}{x-1}$ to answer the following questions.

a. Find $\lim_{x\to 1^+} h(x)$. Show your numerical analysis that leads to your answer and explain what this result implies graphically about $h(x)$ at $x = 1$.

$$\lim_{x\to 1^+} \frac{-2x-\sin(x)}{x-1} = -\infty$$

x	$\frac{-2x-\sin(x)}{x-1}$
1.1	$\frac{-}{+} = -$

$\therefore h(x)$ has a vertical asymptote at $x=1$

b. Find $\lim_{x\to\frac{\pi}{2}}[h(x)\cdot(2x-2)]$. Show your analysis.

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{-2x-\sin x}{\cancel{x-1}}\, 2\cancel{(x-1)}\right)$$

$$= \lim_{x\to\pi/2} (-4x - 2\sin x)$$

$$= -4\left(\tfrac{\pi}{2}\right) - 2\sin\left(\pi/2\right)$$

$$= -2\pi - 2(1)$$

$$= -2\pi - 2$$

c. Explain why the Intermediate Value Theorem guarantees a value of c on the interval [1.5, 2.5] such that $h(c) = -4$. Then, find c.

I. $h(x)$ is continuous on $[1.5, 2.5]$

II. $h(1.5) = -7.995$
$h(2.5) = -3.732$

$$h(1.5) < h(c) = -4 < h(2.5)$$

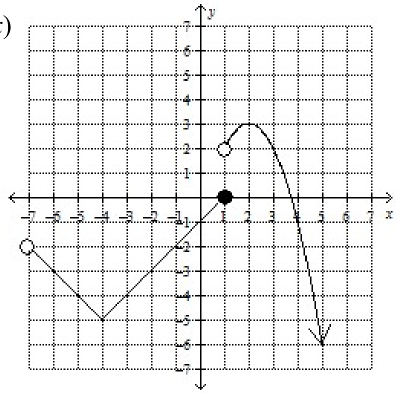
$\therefore$ The IVT guarantees a value of c on $(1.5, 2.5)$ such that $f(c) = -4$.

$$\underbrace{\frac{-2c-\sin c}{c-1}}_{y_1} = \underbrace{-4}_{y_2}$$

$c \approx 2.354$

Free Response Practice #2 Calculator NOT Permitted

Graph of $g(x)$



$$f(x) = \begin{cases} ax + 3, & x < -3 \\ x^2 - 3x, & -3 \le x < 2 \\ bx - 5, & x \ge 2 \end{cases}$$

Pictured above is the graph of a function $g(x)$ and the equation of a piece-wise defined function $f(x)$. Answer the following.

a. Find $\lim\limits_{x \to 1^+} [2g(x) - f(x) \cdot cos(\pi x)]$. Show your work applying the properties of limits.

$= 2\lim\limits_{x\to 1^+} g(x) - \lim\limits_{x\to 1^+} f(x) \cdot \lim\limits_{x\to 1^+} \cos(\pi x)$

$= 2(2) - (1^2 - 3(1)) \cos(\pi(1))$

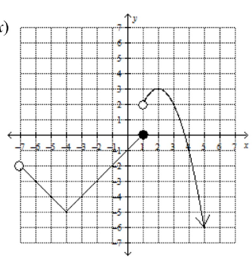
$= 4 - (-2)(-1)$

$= 4 - 2$

$= 2$

b. On its domain, what is one value of x at which $g(x)$ is discontinuous? Use the three part definition of continuity to explain why $g(x)$ is discontinuous at this value.

$g(x)$



I. $g(1) = 0$ $\therefore$ $g(1)$ is defined

II. $\lim\limits_{x\to 1^-} g(x) = 0 \neq \lim\limits_{x\to 1^+} g(x) = 2$

$\therefore$ $\lim\limits_{x\to 1} g(x)$ dne.

$\therefore$ $g(x)$ is not continuous at $x = 1$

c. For what value(s) of a and b, if they exist, would the function $f(x)$ be continuous everywhere? Justify your answer using limits.

$$f(x) = \begin{cases} ax + 3, & x < -3 \\ x^2 - 3x, & -3 \le x < 2 \\ bx - 5, & x \ge 2 \end{cases}$$

$\lim\limits_{x\to -3^-} (ax+3) = \lim\limits_{x\to -3^+} (x^2 - 3x)$

$a(-3)+3 = (-3)^2 - 3(-3)$

$-3a + 3 = 9 + 9$

$-3a + 3 = 18$

$-3a = 15$

$a = -5$

$\lim\limits_{x\to 2^-} (x^2 - 3x) = \lim\limits_{x\to 2^+} (bx - 5)$

$2^2 - 3(2) = b(2) - 5$

$4 - 6 = 2b - 5$

$-2 = 2b - 5$

$3 = 2b$

$3/2 = b$