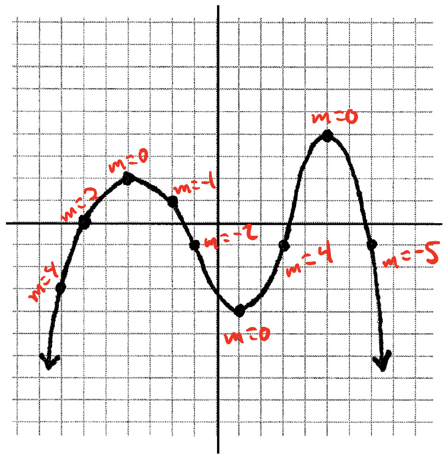
Notes 2.2 Understanding the Derivative from a Graphical and Numerical Approach



So far, our understanding of the derivative is that it represents the slope of the tangent line drawn to a curve at a point.

Complete the table below, estimating the value of $f'(x)$ at the indicated *x* – values by drawing a tangent line and estimating its slope.

***x* – Value**	**Estimation of Derivative**	**Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum**	**Equation of the tangent line at this value of *x*.**
−7	$f'(-7) \approx 4$	Increasing	PoT = (−7, 3) SoT = 4 $y+3=4(x+7)$
−6	$f'(-6) \approx 2$	Increasing	PoT = (−6, 0) SoT = 2 $y-0=2(x+6)$
−4	$f'(-4) \approx 0$	Rel Max	PoT = (−4, 2) SoT = 0 $y-2=0(x+4)$
−2	$f'(-2) \approx -1$	Decreasing	PoT = (−2, 1) SoT = −1 $y-1=-1(x+2)$
−1	$f'(-1) \approx -2$	Decreasing	PoT = (−1, −1) SoT = −2 $y+1=-2(x+1)$
1	$f'(1) \approx 0$	Rel MIN	PoT = (1, 4) SoT = 0 $y-4=0(x-1)$
3	$f'(3) \approx 4$	Increasing	PoT = (3, −1) SoT = 4 $y+1=4(x-3)$
5	$f'(5) \approx 0$	rel max	PoT = (5, 4) SoT = 0 $y-4=0(x-5)$
7	$f'(7) \approx -5$	Decreasing	PoT = (7, −1) SoT = −5 $y+1=-5(x-7)$

Based on what you observed in the table on the previous page, what inferences can you make about the value of the derivative, $f'(x)$, and the behavior of the graph of the function, $f(x)$?

AT any value where $x=a$

- If $f(a)'>0$, then the graph is increasing at $x=a$
- If $f(a)'<0$, then the graph is decreasing at $x=a$
- If $f(a)'=0$, then the graph has a min or max at $x=a$
  - $f(a)$ is a rel max if $f'(x)>0$ on left and $f'(x)<0$ on right of $x=a$.
  - $f(a)$ is a rel min if $f'(x)<0$ on left and $f'(x)>0$ on right of $x=a$.

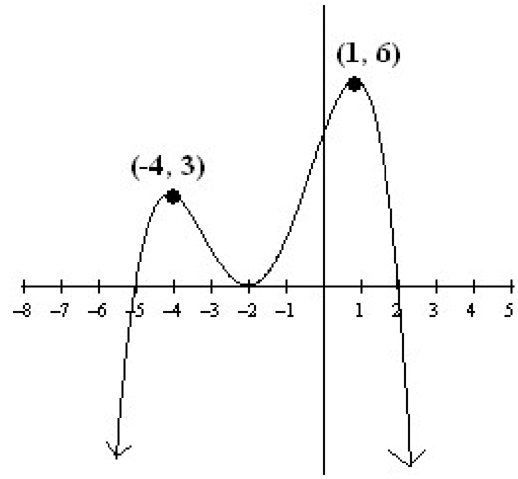
Numerically, the value of the derivative at a point can be estimated by finding the slope of the secant line passing through two points on the graph on either side of the point for which the derivative is being estimated.

x	–3	0	1	4	6	10
***f*(*x*)**	2	1	–3	0	–7	2

***x* – Value**	**Estimation of Derivative** $f'(x) \approx \frac{\Delta y}{\Delta x}$	**Is the function Increasing, Decreasing or at a Relative Maximum or Relative Minimum**	**Equation of the tangent line at this value of *x*.** PoT, SoT; Tangent line $y-y_1=m(x-x_1)$
0	$f'(0) \approx \frac{\Delta y}{\Delta x} \approx \frac{(-3)-(2)}{(1)-(-3)}$, $f'(0) = -5/4$	Decreasing	PoT $=(0,1)$, SoT $\approx -5/4$; $y-1=\frac{-5}{4}(x-0)$
1	$f'(1) \approx \frac{\Delta y}{\Delta x} \approx \frac{(0)-(1)}{(4)-(0)}$, $f'(1) = -1/4$	Decreasing	PoT $=(1,-3)$, SoT $\approx -\frac{1}{4}$; $y+3=\frac{-1}{4}(x-1)$
4	$f'(4) \approx \frac{\Delta y}{\Delta x} \approx \frac{(-7)-(-3)}{(6)-(1)}$, $f'(4) \approx \frac{-4}{5}$	Decreasing	PoT $=(4,0)$, SoT $\approx -4/5$; $y-0=\frac{-4}{5}(x-4)$
6	$f'(6) \approx \frac{\Delta y}{\Delta x} \approx \frac{(2)-(0)}{(10)-(4)} \approx \frac{2}{6}$, $f'(6) \approx 1/3$	Increasing	PoT $=(6,-7)$, SoT $\approx \frac{1}{3}$; $y+7=\frac{1}{3}(x-6)$

The graph of a function, $g(x)$, is pictured to the right. Identify the following characteristics about the graph of the derivative, $g'(x)$. Give a reason for your answers.

The interval(s) where $g'(x) < 0$	$g'(x) < 0$ on $(-4,-2) \cup (1,\infty)$ because $g(x)$ is decreasing.
The interval(s) where $g'(x) > 0$	$g'(x) > 0$ on $(-\infty,-4) \cup (-2,1)$ because $g(x)$ is increasing
The value(s) of x where $g'(x) = 0$	$g'(x) = 0$ when $x = -4, -2,$ and 1 because $g(x)$ has a relative min or max

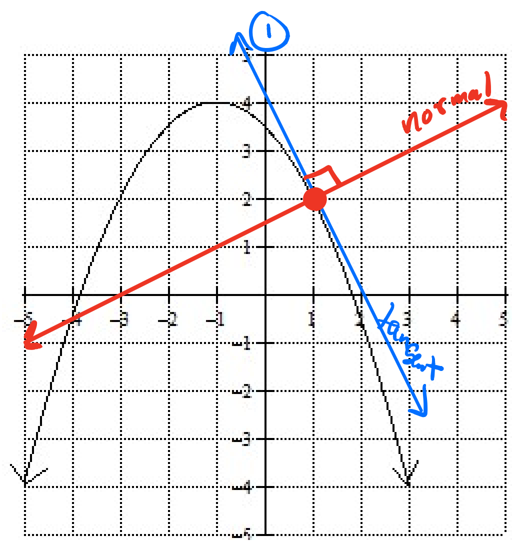


Definition of the Normal Line - The normal line is the line that is perpendicular to the tangent line at the point of tangency.

Pictured to the right is the graph of $f(x) = -\frac{1}{2}(x+1)^2 + 4$.

① Draw the tangent line to the graph of $f(x)$ at $x = 1$. Then, ② $\boxed{\text{estimate}}$ the value of $f'(1)$.

② $f'(1) \approx -2$



③ Find the equation of the tangent line to the graph of $f(x)$ at $x = 1$.

PoT: $(1,2)$ SoT: $f'(1) \approx -2$

Tangent line: $y - y_1 = m(x - x_1)$

③ $y - 2 = -2(x-1)$

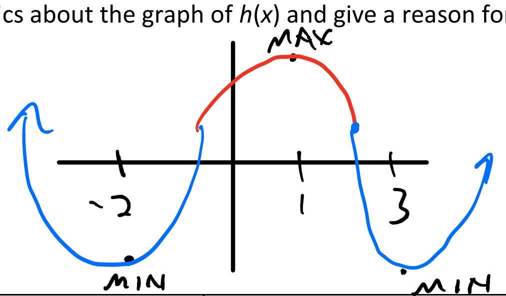
Draw this line and find the equation of the normal line.

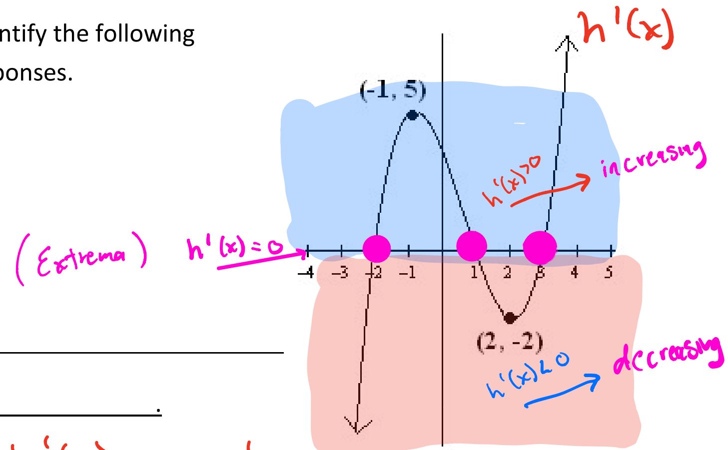
PoT: $(1,2)$ SoN: $f'(1) \approx \frac{1}{2}$

Normal line: $y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x-1)$

The graph of the derivative, $h'(x)$, of a function $h(x)$ is pictured. Identify the following characteristics about the graph of $h(x)$ and give a reason for your responses.





The interval(s) where $h(x)$ is increasing	If h(x) is increasing, then $h'(x)$ > 0. If h' > 0, then h'(x) is above the x-axis. Therefore h(x) is increasing on (-2, 1) ∪ (3, ∞)
The interval(s) where $h(x)$ is decreasing	If h(x) is decreasing, then $h'(x)$ < 0. If h' < 0, then h' is below the x-axis. Therefore h(x) is decreasing on (-∞, -2) ∪ (1, 3)
The value(s) of x where $h(x)$ has a relative maximum.	If h(x) is an extrema, then $h'(x)$ = 0, or on the x-axis If h(x) is a relative maximum, then $h'(x)$ changes from positive to negative which means it goes from above to below the x-axis. Therefore h(x) has a relative maximum at x = 1
The value(s) of x where $h(x)$ has a relative minimum. POT/PON	If h(x) is an extrema, then $h'(x)$ = 0, or on the x-axis If h(x) is a relative minimum, then $h'(x)$ changes from negative to positive which means it goes from below to above the x-axis. Therefore h(x) has a relative minimum at x = -2, 3
If $h(-1) = \frac{1}{2}$, what is the equation of the tangent line drawn to the graph of $h(x)$ at $x = -1$?	POT/PON = $(-1, \frac{1}{2})$ (x, h(x)) SOT = 5 because (-1, 5) (x, h'(x)) Tangent line $y - y_1 = m(x - x_1)$ $y - \frac{1}{2} = 5(x + 1)$
If $h(2) = -3$, what is the equation of the normal line drawn to the graph of $h(x)$ at $x = 2$?	POT/PON = (2, -3) SOT = -2 Tangent line $y - y_1 = m(x - x_1)$ $y + 3 = -2(x - 2)$