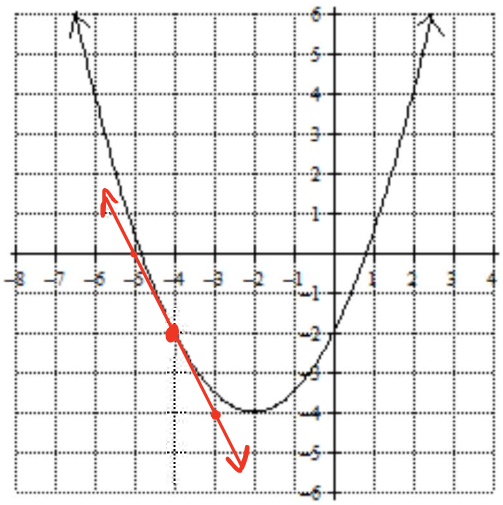
Notes 2.5 – Solidifying the Derivative as the Tangent Line

Pictured is the graph of $g(x) = \frac{1}{2}(x+2)^2 - 4$.



1. Find $g'(-4)$ and explain what this value represents in terms of the graph of the function $g(x)$.

$g(x) = \frac{1}{2}(x^2 + 4x + 4) - 4$

$= \frac{1}{2}x^2 + 2x + 2 - 4$

$g(x) = \frac{1}{2}x^2 + 2x - 2$

$g'(x) = x + 2$

$g'(-4) = (-4) + 2$

$\boxed{g'(-4) = -2}$

$g'(-4) = -2$ represents the slope of the tangent line of $g(x)$ at $x = -4$

2. Find the equation of the tangent line drawn to the graph of $g(x)$ at $x = -4$. Sketch a graph of this tangent line on the grid with the graph of $g(x)$ above.

PoT $= (-4, -2)$

SoT $= -2$

$y + 2 = -2(x + 4)$

3. Using the equation of the tangent line, find the value of y when $x = -3.9$. Then, find the value of $g(-3.9)$.

$y + 2 = -2(x + 4)$

$y + 2 = -2[(-3.9) + 4]$

$y + 2 = -2[0.1]$

$y + 2 = -0.2$

$y = -2.2$

$g(x) = \frac{1}{2}(x + 2)^2 - 4$

$g(-3.9) = \frac{1}{2}(-3.9 + 2)^2 - 4$

$= \frac{1}{2}(-1.9)^2 - 4$

$g(-3.9) = -2.195$

4. What do you notice about the values of these two results from question 3? What does this imply about how the equation of the tangent line might be used?

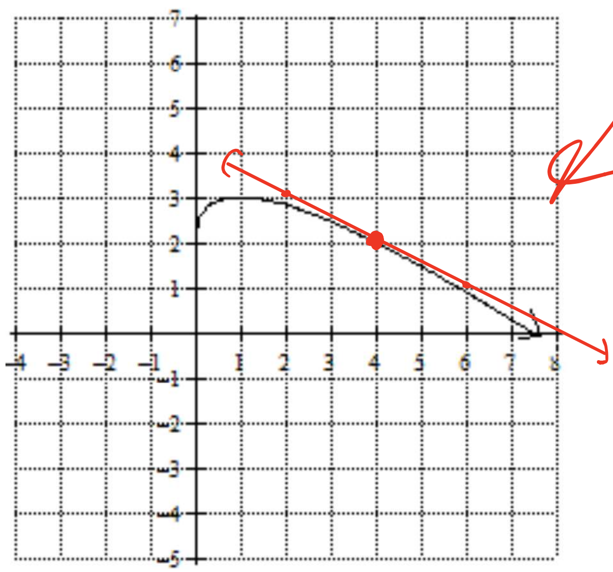
The two results are very close in value to each other. on this bright, Sunny Thur—.

The equation of the tangent line can be used to approximate a value of the function to the point of tangency.

$g'(x) = 3x^2 - 2x + 8$

$g(x) = x^3 - x^2 + 8x + C$

Pictured to the right is function $g(x) = 2\sqrt{x} - x + 2$. Use the graph and the equation to answer 5 – 9.



5. Based on the graph, at what value(s) does the graph of $g(x)$ have a horizontal tangent? Give a reason. Show an algebraic analysis that supports your answer.

g(x) appears to have a horizontal tangent at $x = 1$ b/c g(x) has a maximum there.

$g(x) = 2 \cdot x^{1/2} - x + 2$

$\frac{dg}{dx} = x^{-1/2} - 1$

$0 = \frac{1}{\sqrt{x}} - 1$

$1 = \frac{1}{\sqrt{x}}$

$\sqrt{x} = 1$

$x = 1$

Since $g'(x) = 0$ when $x = 1$, then $g(x)$ has a horizontal tangent at $x = 1$

6. On what interval(s) is $g'(x) < 0$? Give a reason for your answer.

$g'(x) < 0$ on $(1, \infty)$ b/c $g(x)$ is decreasing on $(1, \infty)$

7. On what interval(s) is $g'(x) > 0$? Give a reason for your answer.

$g'(x) > 0$ on $(0, 1)$ b/c $g(x)$ is increasing on that interval.

8. For what value(s) of x is the slope of the tangent line equal to 2? Show your work.

$g'(x) = \frac{1}{\sqrt{x}} - 1$

$2 = \frac{1}{\sqrt{x}} - 1$

$3 = \frac{1}{\sqrt{x}}$

$3\sqrt{x} = 1$

$\sqrt{x} = \frac{1}{3}$

$x = \frac{1}{9}$

9. Find an equation of the tangent line drawn to the graph of $g(x)$ when $x = 4$. Then, draw the tangent line on the grid above.

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 4)$

PoT = $(4, 2)$

SoT = $-\frac{1}{2}$

$g'(4) = \frac{1}{\sqrt{4}} - 1$

$= \frac{1}{2} - 1$

$g'(4) = -\frac{1}{2}$

The table of values below represents values on the graph of the derivative, $h'(x)$, of a polynomial function $h(x)$. The zeros indicated in the table are the only zeros of the graph of $h'(x)$. Additionally, the graph of $h(x)$ is concave up at $x = 3$. Use the table to answer questions 10 – 15.

x	−8	−5	−2	0	3	5	7	10	12
h′(x)	11	5	0	−1	−3	−1	0	−3	−9

10. On what interval(s) is the function $h(x)$ increasing and decreasing? Give reasons for your answers.

$h(x)$ is increasing on $(-\infty, -2)$ b/c $h' > 0$
$h(x)$ is decreasing on $(-2, 7) \cup (7, \infty)$ b/c $h' < 0$

11. At what x – value(s) does the graph of $h(x)$ have a relative maximum? Justify your answer.

$h(x)$ has a relative max at $x = -2$
b/c $h'(x)$ changes from positive
to negative.

12. At what x – value(s) does the graph of $h(x)$ have a relative minimum? Justify your answer.

$h(x)$ has no mins b/c
$h'(x)$ never changes from negative
to positive.

13. If $h(3) = 2$, what is the equation of the tangent line to the graph of $h(x)$ at $x = 3$? What is the equation of the normal line to the graph of $h(x)$ at $x = 3$?

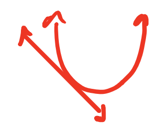
PoT = (3,2)
SoT = $h'(3) = -3$

$y - 2 = -3(x-3)$ tangent
$y - 2 = \frac{1}{3}(x-3)$ normal

14. Find the tangent line approximation of $h(3.1)$. Is this approximation greater or less than the actual value of $h(3.1)$? Give a reason for your answer.

$$\boxed{\begin{aligned} y-2 &= -3(x-3)\\ y-2 &= -3[3.1-3]\\ y-2 &= -3[0.1]\\ y-2 &= -0.3\\ y &= 1.7\end{aligned}}$$

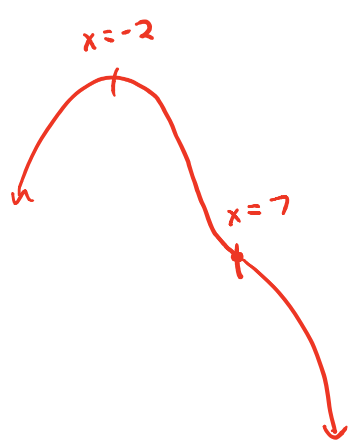
$h(3.1) \approx 1.7$



$h(3.1) \approx 1.7$ is less than the actual value of $h(3.1)$ b/c $h(x)$ is concave up at $x = 3$.

15. Find the value of each of the following limits:

$\lim_{x\to-\infty} h(x) = -\infty$ $\lim_{x\to\infty} h(x) = -\infty$



The derivative of a polynomial function, $f(x)$, is given by the equation $f'(x) = x(2-x)(x+3)$. Use this equation to answer questions 16 – 20.

16. On what intervals is $f(x)$ increasing? Decreasing? Justify your answers.

$f'(x)=0$ when $x=-3,0,2$

~~$f'(x) = x(2-x)(x+3)$~~

~~$0 = x(2-x)(x+3)$~~

~~$0=x \quad 2-x=0 \quad x+3=0$~~

~~$x=2 \quad x=-3$~~

$(-)(+)(-) = +$ | $(-)(+)(+) = -$ | $(+)(+)(+) = +$ | $(+)(-)(+) = -$

$f'(x)$: positive -3 neg 0 pos 2 neg

- $f(x)$ is increasing on $(-\infty,-3)\cup(0,2)$ b/c $f'>0$ on these intervals.
- $f(x)$ is decreasing $(-3,0)\cup(2,\infty)$ b/c $f'<0$ on these intervals

17. At what value(s) of x does the graph of $f(x)$ reach a relative minimum? Justify your answers.

$f(x)$ has a rel min at $x=0$ b/c $f'(x)$ changes from neg. to pos.

18. At what value(s) of x does the graph of $f(x)$ reach a relative maximum? Justify your answers.

$f(x)$ has a rel. max. at $x=-3, 2$ b/c $f'(x)$ changes from pos. to neg.

19. If $f(4) = -1$, what is the equation of the tangent line drawn to the graph of $f(x)$ at $x = 4$?

PoT $= (4,-1)$

SoT $= -56$

$y+1=-56(x-4)$

$$\boxed{\begin{aligned} f'(4) &= 4(2-4)(4+3) \\ &= 4(-2)(7) \\ f'(4) &= -56 \end{aligned}}$$

20. Approximate the value of $f(4.1)$. If $f(x)$ is concave down at $x = 4$, is this an over or under approximation of $f(4.1)$? Explain your reasoning.

$y+1=-56(x-4)$

$y+1=-56((4.1)-4)$

$y+1=-56(0.1)$

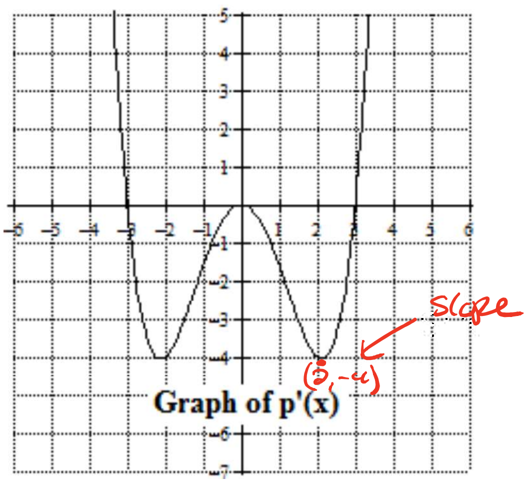
$y+1=-5.6$

$y=-6.6$

$\boxed{\therefore f(4.1)\approx -6.6}$

$f(4.1)\approx -6.6$ is an over approximation of $f(4.1)$ b/c $f(x)$ is concave down at $x=4$.

Pictured to the right is a graph of $p'(x)$, the derivative of a polynomial function, $p(x)$. Use the graph to answer the questions 21 – 25.



21. On what interval(s) is the graph of $p(x)$ decreasing? Justify your answer.

$p(x)$ is decreasing on $(-3,0) \cup (0,3)$

b/c $p'(x) < 0$ on these intervals.

22. On what interval(s) is the graph of $p(x)$ increasing? Justify your answer.

$p(x)$ is increasing on $(-\infty,-3) \cup (3,\infty)$

b/c $p'(x) > 0$ on these intervals.

23. At what value(s) of x does the graph of $p(x)$ reach a relative maximum? Justify your answer.

There is a rel max on $p(x)$ at $x=-3$

b/c $p'(x)$ changes from pos to neg at $x=-3$

24. At what value(s) of x does the graph of $p(x)$ reach a relative minimum? Justify your answer.

$p(x)$ reaches a rel min at $x=3$

b/c $p'(x)$ changes from neg to pos at $x=3$

25. Approximate the value of $p(1.8)$ using the tangent line approximation if $p(2) = -3$.

PoT: $(2,-3)$

SoT $= -4 = p'(2)$ graph

$y+3=-4(x-2)$

$y+3=-4(1.8-2)$

$y+3=-4(-0.2)$

$y+3= 0.8$

$y=-2.2$

$p(1.8) \approx -2.2$