Notes 4.1 – Implicit Differentiation

Explicit function vs implicit function

Find $\frac{d}{dx}$ using explicit differentiation.

$$x^2 + y^2 = 25$$

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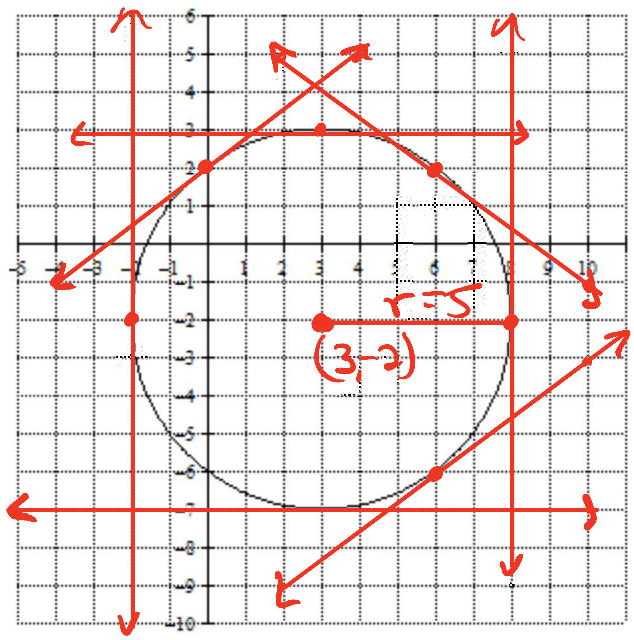
Explicit	Implicit
$x^2 + y^2 = 25$	$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}25$
$y^2 = 25 - x^2$	$2x + 2y\frac{dy}{dx} = 0$
$y = \pm\sqrt{25 - x^2}$	$2y\frac{dy}{dx} = -2x$
$y = \pm\sqrt{25 - x^2}$	$\frac{dy}{dx} = \frac{-x}{y}$
$\frac{dy}{dx} = \pm\frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)$	
$\frac{dy}{dx} = \frac{-x}{\pm\sqrt{25 - x^2}}$	

Consider the graph of the circle to the right. Find the equation of the circle in implicit form below.

$(x-3)^2+(y+2)^2=25$



Now, implicitly differentiate the equation of the circle in the space below. with respect to x

$\frac{d}{dx}(x-3)^2+\frac{d}{dx}(y+2)^2=\frac{d}{dx}25$

$2(x-3)(1)+2(y+2)(1)\frac{dy}{dx}=0$

$2(y+2)\frac{dy}{dx}=-2(x-3)$

$\frac{dy}{dx}=\frac{-2(x-3)}{2(y+2)}$

$\frac{dy}{dx}=-\frac{x-3}{y+2}$

Complete the table below finding the value of $\frac{dy}{dx}$ at each of the indicated points. Then, draw the graphical representation, the tangent line, on the graph at each indicated point.

(0, 2)	$\left.\frac{dy}{dx}\right\|_{(0,2)}=-\frac{0-3}{2+2}=\frac{3}{4}$
(3, 3)	$\left.\frac{dy}{dx}\right\|_{(3,3)}=-\frac{3-3}{3+2}=\frac{-0}{5}=0$ Horizontal tangent
(8, –2)	$\left.\frac{dy}{dx}\right\|_{(8,-2)}=-\frac{8-3}{-2+2}=\frac{-5}{0}=$ UNDIES vertical tangent
(3, –7)	$\left.\frac{dy}{dx}\right\|_{(3,-7)}=-\frac{3-3}{-7+2}=\frac{0}{-5}=0$
(6, 2)	$\left.\frac{dy}{dx}\right\|_{(6,2)}=-\frac{6-3}{2+2}=\frac{-3}{4}$
(–2 , –2)	$\left.\frac{dy}{dx}\right\|_{(-2,-2)}=-\frac{-2-3}{-2+2}=\frac{5}{0}=$ UNDIES
(6, –6)	$\left.\frac{dy}{dx}\right\|_{(6,-6)}=-\frac{6-3}{-6+2}=\frac{-3}{-4}=\frac{3}{4}$

Find $\frac{dy}{dx}$ for each of the following implicitly defined equations.

$$y^2 = 2x + 3y$$

$$\frac{d}{dx}y^2 = \frac{d}{dx}2x + \frac{d}{dx}3y$$

$$2y\frac{dy}{dx} = 2 + 3\frac{dy}{dx}$$

$$2y\frac{dy}{dx} - 3\frac{dy}{dx} = 2$$

$$\frac{dy}{dx}(2y-3) = 2$$

$$\frac{dy}{dx} = \frac{2}{2y-3}$$

$$\frac{d}{dx}2x + \frac{d}{dx}e^y = \frac{d}{dx}x^2 - \frac{d}{dx}y^2$$

$$2 + e^y\frac{dy}{dx} = 2x - 2y\frac{dy}{dx}$$

$$e^y\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 2$$

$$\frac{dy}{dx}(e^y + 2y) = 2x - 2$$

$$\frac{dy}{dx} = \frac{2x-2}{e^y+2y}$$

$$\frac{d}{dx}\underbrace{2xy}_{\text{Product}} = \frac{d}{dx}3y^2 + \frac{d}{dx}2x$$

$$\overbrace{2\cdot y + 2x\cdot(1)\frac{dy}{dx}}^{\text{PRODUCT}} = 6y\frac{dy}{dx} + 2$$

$$2x\frac{dy}{dx} - 6y\frac{dy}{dx} = 2 - 2y$$

$$\frac{dy}{dx}(2x - 6y) = 2 - 2y$$

$$\frac{dy}{dx} = \frac{2-2y}{2x-6y}$$

$$\frac{dy}{dx} = \frac{2(1-y)}{2(x-3y)}$$

$$\frac{dy}{dx} = \frac{1-y}{x-3y}$$

$$\frac{d}{dx}5x^3 + \frac{d}{dx}3 = \frac{d}{dx}2y - \frac{d}{dx}\underbrace{3x^2y}_{\text{Product}}$$

$$15x^2 + 0 = 2\frac{dy}{dx} - \overbrace{\left[6x\cdot y + 3x^2\cdot(1)\frac{dy}{dx}\right]}^{\text{Product}}$$

$$15x^2 = 2\frac{dy}{dx} - 6xy + 3x^2\frac{dy}{dx}$$

$$15x^2 + 6xy = 2\frac{dy}{dx} + 3x^2\frac{dy}{dx}$$

$$15x^2 + 6xy = \frac{dy}{dx}(2 + 3x^2)$$

$$\frac{15x^2+6xy}{2+3x^2} = \frac{dy}{dx}$$

CALC

$\frac{dy}{dx}=0$

For what values of x will the curve $x^3+y^3=4xy+1$ have a horizontal tangent? Show your work and explain your thinking.

Product

$\frac{d}{dx}x^3+\frac{d}{dx}y^3=\frac{d}{dx}4xy+\frac{d}{dx}1$

$3x^2+3y^2\frac{dy}{dx}=4\cdot y+4x\cdot(1)\frac{dy}{dx}+0$

$3y^2\frac{dy}{dx}-4x\frac{dy}{dx}=4y-3x^2$

$\frac{dy}{dx}(3y^2-4x)=4y-3x^2$

$\frac{dy}{dx}=\frac{4y-3x^2}{3y^2-4x}$

$0=\frac{4y-3x^2}{3y^2-4x}$

(NEED Another EQ)

$\boxed{\begin{array}{l}0=4y-3x^2\\ x^3+y^3=4xy+1\end{array}}$

$4y=3x^2$

$y=\frac{3x^2}{4}$

$x^3+\left(\frac{3x^2}{4}\right)^3=4x\left(\frac{3x^2}{4}\right)+1$

$x^3+\frac{27x^6}{64}=3x^3+1$

$\frac{27}{64}x^6-2x^3-1=0$

$x\approx-0.770,\ 1.732$

In terms of y, describe the values of x for which the curve $x^3+y^3=4xy+1$ will have a vertical tangent? Show your work and explain your thinking.

The curve has a vertical tangent when $\frac{dy}{dx}$ is undefined

$0=3y^2-4x$

$4x=3y^2$

$x=\frac{3}{4}y^2$

Given the curve $y^2+2y=2x+1$, find $\frac{d^2y}{dx^2}$.	Given the curve $x^2+y^2=1$, find $\frac{d^2y}{dx^2}$.
$2y\frac{dy}{dx}+2\frac{dy}{dx}=2$ $\frac{dy}{dx}(2y+2)=2$ $\frac{dy}{dx}=\frac{2}{2y+2}$ $\frac{dy}{dx}=\frac{1}{y+1}$ $\frac{d^2y}{dx^2}=\frac{(0)\cdot(y+1)-1(1)\frac{dy}{dx}}{(y+1)^2}$ $\frac{d^2y}{dx^2}=\frac{-\left(\frac{1}{y+1}\right)}{(y+1)^2}\cdot\frac{(y+1)}{(y+1)}$ $\frac{d^2y}{dx^2}=\frac{-1}{(y+1)^3}$	$2x+2y\frac{dy}{dx}=0$ $2y\frac{dy}{dx}=-2x$ $\frac{dy}{dx}=\frac{-x}{y}$ $\frac{dy^2}{dx^2}=\frac{-1\cdot y-(-x)\cdot(1)\frac{dy}{dx}}{y^2}$ $\frac{dy^2}{dx^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}$ $\frac{dy^2}{dx^2}=\frac{-y-\frac{x^2}{y}}{y^2}=\frac{\frac{-y^2-x^2}{\cancel{y}}}{y^2}\ \frac{y}{y}$ $\frac{dy^2}{dx^2}=-\frac{y^2+x^2}{y^3}$