Notes 4.5 – Analytical and Graphical Connections between $f(x)$, $f'(x)$, and $f''(x)$ – Part II

Since the SECOND DERIVATIVE is the FIRST DERIVATIVE of $F'(x)$, then the same relationships that exist between $F(x)$ and $F'(x)$ must exist between $F'(x)$ and $F''(x)$.

With these relationships in mind, complete the following table.

If $F''(x)$…	Then $F(x)$…	And $F'(x)$…
…is = 0 or is undefined at $x = a$,	has a potential Point of inflection	has a potential rel max or min
…is > 0,	is concave up	is increasing
…is < 0,	is concave down	is decreasing
…changes from positive to negative,	has an IP	has a rel max
…changes from negative to positive,	has an IP	has a rel min

Consider the function $h'(x) = 2x - x\sin(2x)$ on the open interval $-5 < x < 5$ to answer the following questions.

1. Based on the graph of $h'(x)$, how many relative extrema does the graph of $h(x)$ have? Give a reason for your answer.

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Since $h'(x)$ has one solution, we expect 1 relative extrema on $h(x)$

2. Based on the graph of $h'(x)$, how many points of inflection does the graph of $h(x)$ have? Give a reason for your answer.

$h(x)$ has 5 inflection points because $h'(x)$ has 5 relative extrema on $(-5, 5)$

3. Find the equation of $h''(x)$ and then graph it on your calculator. Explain why the graph of $h''(x)$ confirms your response to question #2 above.

$h''(x) = 2 - [1 \cdot \sin(2x) + x\cos(2x) \cdot 2]$

$0 = 2 - \sin 2x + 2x\cos 2x$

$h''(x) = 0$ on five location on the x-axis on $(-5, 5)$

Based on these relationships between a function and its first and second derivative, complete the following statements.

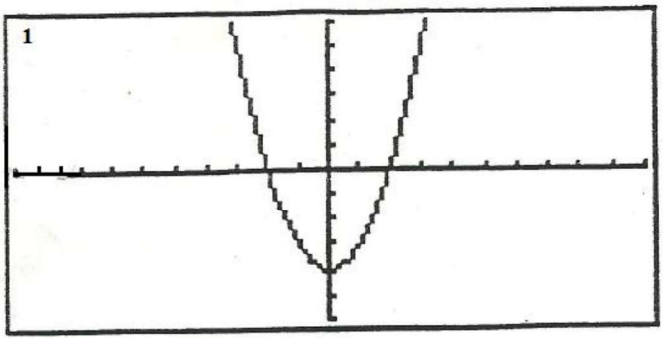
1. $f(x)$ is increasing $\leftrightarrow$ $f'(x)$ >0
2. $f(x)$ is decreasing $\leftrightarrow$ $f'(x)$ <0
3. $f(x)$ has a relative maximum or minimum $\leftrightarrow$ $f'(x)$ $=0$ or und
4. $f(x)$ has a point of inflection $\leftrightarrow$ $f'(x)$ has an extrema
5. $f(x)$ is concave up $\leftrightarrow$ $f''(x)$ >0
6. $f(x)$ is concave down $\leftrightarrow$ $f''(x)$ <0
7. $f(x)$ has a point of inflection $\leftrightarrow$ $f''(x)$ $=0$ or und
8. $f'(x)$ is increasing $\leftrightarrow$ $f''(x)$ >0
9. $f'(x)$ is decreasing $\leftrightarrow$ $f''(x)$ <0
10. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f''(x)$ $=0$ or und
11. $f'(x)$ has a point of inflection $\leftrightarrow$ $f''(x)$ has an extrema
12. $f'(x)$ changes from negative to positive $\leftrightarrow$ $f(x)$ has a min
13. $f'(x)$ changes from positive to negative $\leftrightarrow$ $f(x)$ has a max
14. $f'(x)$ has a relative maximum or minimum $\leftrightarrow$ $f(x)$ has an IP
15. $f''(x)$ changes from positive to negative $\leftrightarrow$ $f'(x)$ has a max
16. $f''(x)$ changes from negative to positive $\leftrightarrow$ $f'(x)$ has a min

On the following pages, you will find 15 graphs of polynomial functions. The 15 graphs can be put into five groups of three so that the three graphs in each group would represent $f(x)$, $f'(x)$, and $f''(x)$. Once you have the graphs grouped, record the groups in the table to the right using the numbers 1 through 15.

This works best if the graphs are cut out so that students can arrange them in the different groups.

	$f(x)$	$f'(x)$	$f''(x)$
1.	2	13	7
2.	4	5	8
3.	9	12	3
4.	10	1	11
5.	15	6	14

1



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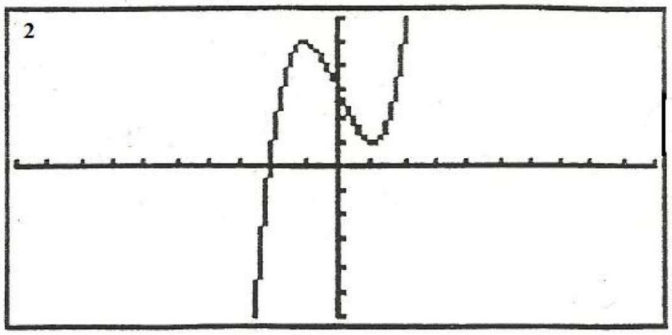

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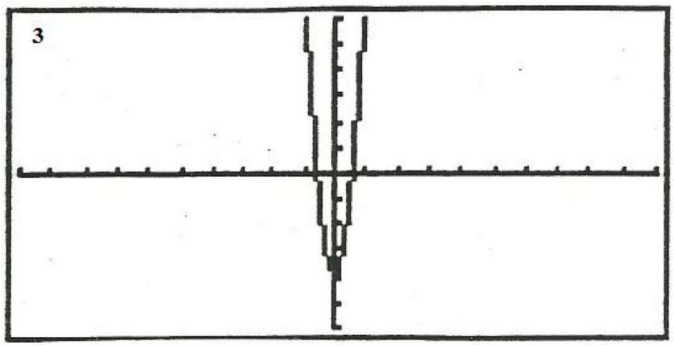

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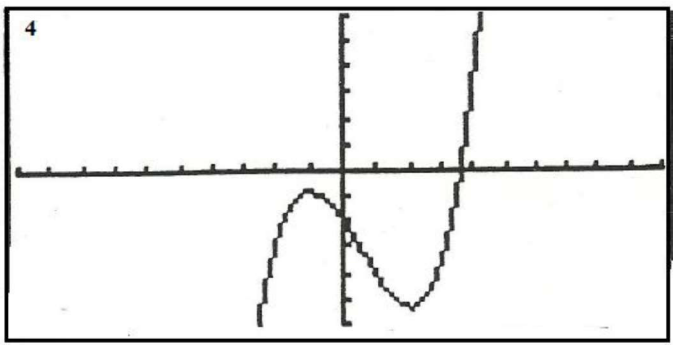

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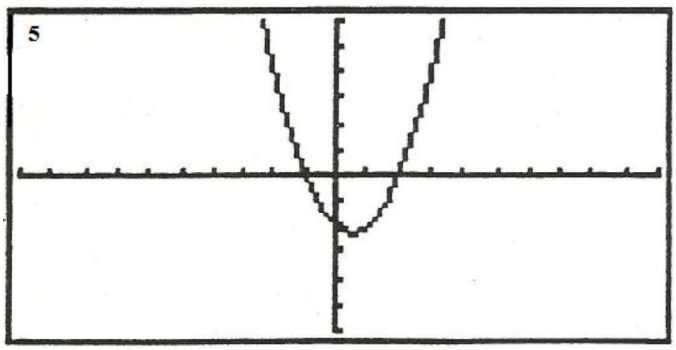

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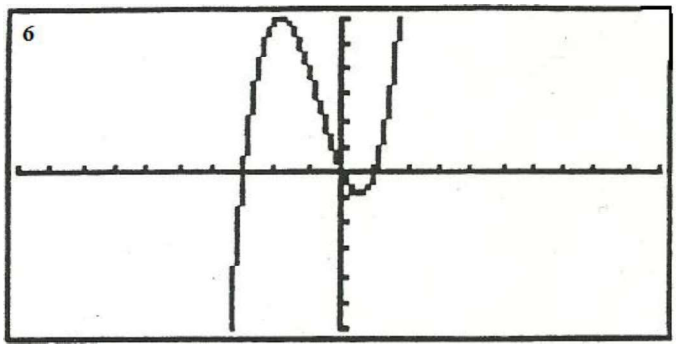

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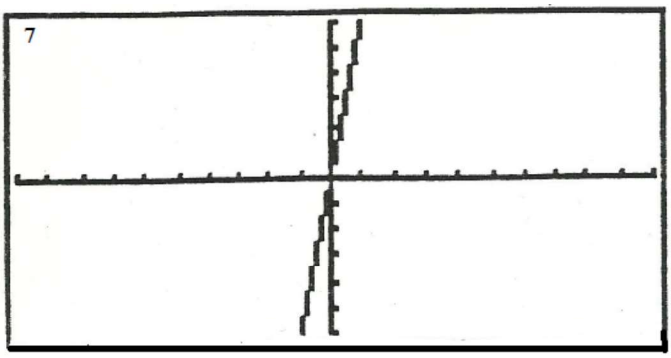

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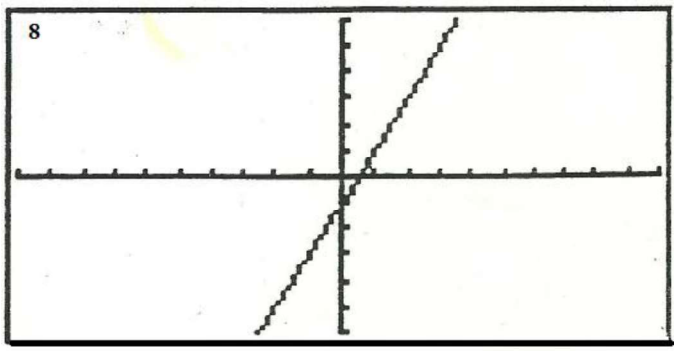

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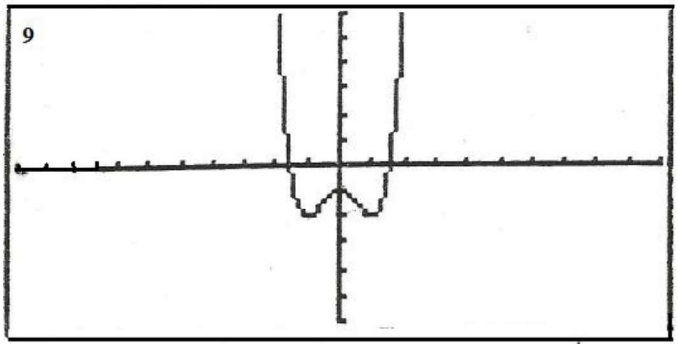

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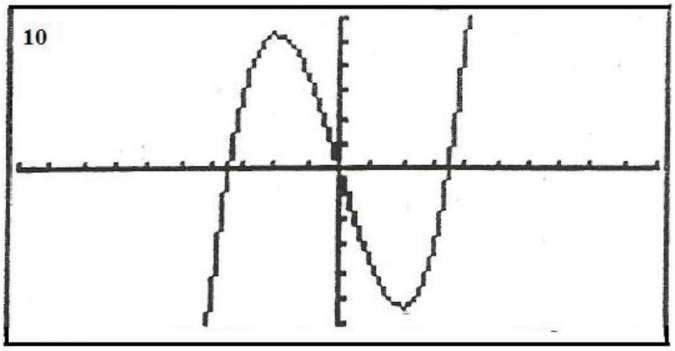

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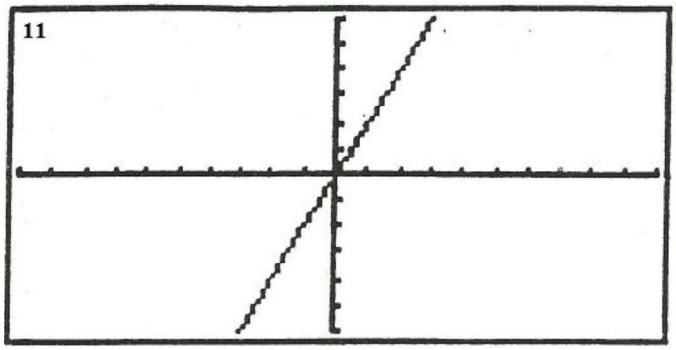

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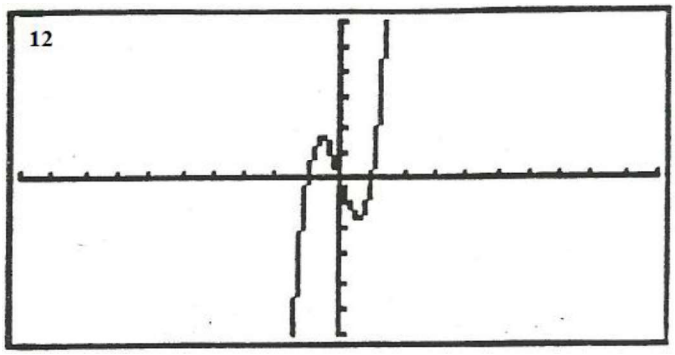

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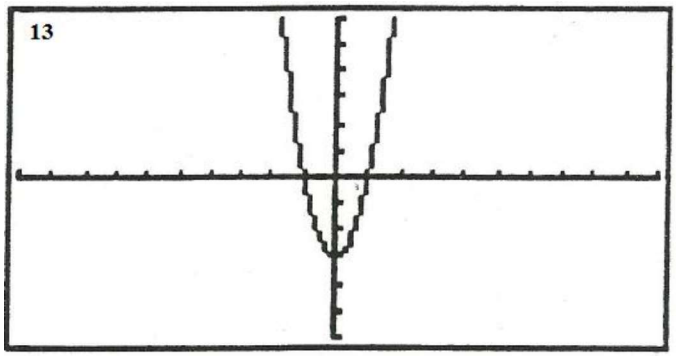

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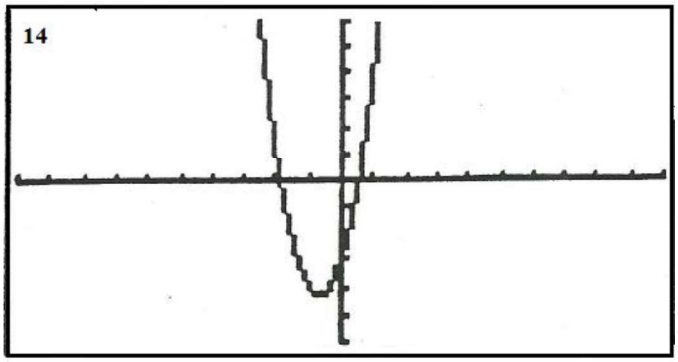

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