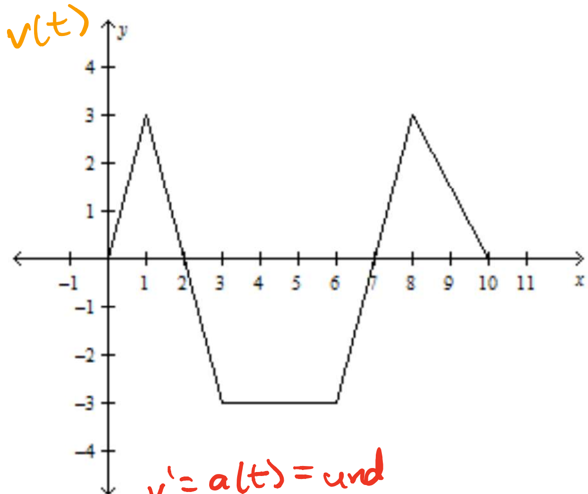
Notes 5.5 – More on Particle Motion
Finding Net and Total Distance

The graph below represents the velocity, $v(t)$ which is measured in meters per second, of a particle moving along the x – axis.



$v' = a(t) = \text{und}$

At what value(s) of t does the particle have no acceleration on the interval (0, 10)? Justify your answer.

$a(t)$ is undefined at $t = 1, 3, 6$ and 8 on $(0, 10)$ b/c the graph of $v(t)$ has cusps at these t-values.

Express the acceleration, $a(t)$, as a piecewise-defined function on the interval (0, 10).

$$a(t) = \begin{cases} 3, & 0 < t < 1 \\ -3, & 1 < t < 3 \\ 0, & 3 < t < 6 \\ 3, & 6 < t < 8 \\ -3/2, & 8 < t < 10 \end{cases}$$

For what value(s) of t is the particle moving to the right? To the left? Justify your answer.

- The particle is moving to the right on $0 < t < 2$ and $7 < t < 10$ b/c $v(t) > 0$ on these intervals.
- The particle is moving to the left on $2 < t < 7$ b/c $v(t) < 0$ on these intervals.

Find the average acceleration of the particle on the interval [1, 8]. Show your work.

$$\text{Average Acceleration} = \frac{v(1) - v(8)}{1 - 8}$$
$$= \frac{3 - 3}{-7}$$
$$= \frac{0}{-7}$$
$$= 0 \text{ meters/second}^2$$

Definition of Net Distance: The distance between the point of origin and the final position

(Displacement)

Definition of Total Distance: The sum of all the distances moved in any direction

If a particle is moving in the same direction the entire amount of time, what can be said about the net distance and the total distance?

The net distance = The total distance

To Find the Net Distance a Particle Travels on an Interval

Net Distance $= |p(a) - p(b)|$ on $[a, b]$

To Find the Total Distance a Particle Travels on an Interval

Total Distance $= |p(a) - p(c)| + |p(c) - p(b)|$ on $[a, b]$

where $t = c$ is when the particle changes direction

CALC The position of a particle is given by the function $p(t) = 2t^3 - 6t^2 + 8t$ where $p(t)$ is measured in centimeters. Find the net and total distance the particle travels from $t = 1.5$ seconds to $t = 4$ seconds.

$\boxed{\text{Net Distance} = |p(1.5) - p(4)| = 58.75 \text{ cm}}$

Does the particle change direction on $(1.5, 4)$

$v(t) = p'(t) = 6t^2 - 12t + 8$

$0 = 6(t^2 - 2t + 1) + 8 - 6$

$0 = 6(t - 1)^2 + 2$

$-\frac{2}{6} = (t - 1)^2$

$\pm\sqrt{-\frac{2}{6}} = t - 1 \rightarrow t = \text{UND}$

The total distance is also 58.75 cm because the particle is always moving right on $(1.5, 4)$ because $v(t) > 0$ on $(1.5, 4)$

CALC

The position of a particle is given by the function $p(t) = e^{2t} - 8t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = 0.5$ minutes to $t = 1.5$ minutes.

$\boxed{\text{Net Distance} = |p(0.5) - p(1.5)| = 9.367 \text{ feet}}$

Change directions on $(0.5, 1.5)$?

$v(t) = 2e^{2t} - 8$

$0 = 2e^{2t} - 8$

$8 = 2e^{2t}$

$4 = e^{2t}$

$\ln 4 = 2t$

$\frac{1}{2}\ln 4 = t$

$0.693 \approx t$

The particle changes direction at $t = 0.693$

$v(t)$: NEG on (0.5, 0.693), POS on (0.693, 1.5)

0.5 0.693 1.5

Since $v(t)$ changes signs, $v(t)$ changes directions. Therefore net $\neq$ total

$$\text{Total Distance} = |p(0.5) - p(0.693)| + |p(0.693) - p(1.5)| \approx 9.894 \text{ ft}$$

The position of a particle is given by the function $p(t) = t + 2\sin t$ where $p(t)$ is measured in feet. Find the net and total distance the particle travels from $t = \frac{\pi}{6}$ minutes to $t = \frac{5\pi}{4}$ minutes.

$$\text{Net Distance} = \left| p\left(\tfrac{\pi}{6}\right) - p\left(\tfrac{5\pi}{4}\right) \right| = 0.989 \text{ feet}$$

Change Direction?

$v(t) = 1 + 2\cos t$

$0 = 1 + 2\cos t$

$-1 = 2\cos t$

$-\frac{1}{2} = \cos t$

$t = \frac{2\pi}{3}, \cancel{\frac{4\pi}{3}}$

$\therefore$ The particle changes direction at $t = 2\pi/3$

$v(t)$: $\pi/6$ $2\pi/3$ $5\pi/4$

$$\text{Total Distance} = \left| p(\pi/6) - p\left(\tfrac{2\pi}{3}\right) \right| + \left| p(2\pi/3) - p\left(\tfrac{5\pi}{4}\right) \right| \approx 3.617 \text{ feet}$$