

Notes 6.1 – Finding Anti-derivatives of Polynomial-Type Functions

Power Rule for Integration

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

where $n \neq -1$

Find each of the following anti-derivatives.

1. $\int (3x^2 + 2x + 3)dx$

$$= x^3 + x^2 + 3x + C$$

2. $\int \left(\frac{x^3 + 2x^2 - 4x}{x} \right) dx = \int (x^2 + 2x - 4) dx$

$$= \frac{1}{3}x^3 + x^2 - 4x + C$$

3. $\int (x+2)(2x-3)dx$

$$= \int (2x^2 + 4x - 3x - 6)dx$$

$$= \int (2x^2 + x - 6)dx$$

$$= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 6x + C$$

4. $\int \frac{2}{\sqrt{x}} dx = \int 2x^{-1/2} dx$
 $= 2 \cdot 2x^{1/2} + C$
 $= 4\sqrt{x} + C$

5. $\int \frac{1}{x} dx = \int x^{-1} dx = \frac{1}{0} x^0 + C$

Undefined if we try to use
the power rule. But, there is
another rule . . .

Integration Rule for $\frac{1}{x}$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Integration Rules for sine and cosine

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

Use the given information about f' and f'' to find $f(x)$.

10. $f''(x) = 2$ $f'(2) = 5$ $f(2) = 10$

$$f'(x) = \int f''(x) dx = \int 2 dx = 2x + C$$

$$f'(c) = 2x + C$$

① $f'(2) = 5$

$$5 = 2(2) + C$$

$$5 = 4 + C$$

$$1 = C$$

$$\therefore f'(x) = 2x + 1$$

Find each of the following anti-derivatives.

6. $\int (2 \sin x + \cos x) dx = -2 \cos x + \sin x + C$

7. $\int \left(\frac{4}{x} - 3 \cos x\right) dx = 4 \ln|x| - 3 \sin x + C$

8. $\int (t^2 - \sin t) dt = \frac{1}{3}t^3 + \cos t + C$

9. $\int (\sqrt{x} + \sin x) dx = \frac{2}{3}x^{\frac{3}{2}} - \cos x + C$

$$f(x) = \int f'(x) dx = \int (2x + 1) dx$$

$$f(x) = x^2 + x + C$$

② $f(2) = 10$

$$10 = (2)^2 + (2) + C$$

$$10 = 4 + 2 + C$$

$$4 = C$$

$$\boxed{\therefore f(x) = x^2 + x + 4}$$

Use the given information about f' and f'' to find $f(x)$.

$$11. f''(x) = x^{-3/2} \quad f'(4) = 2 \quad f(0) = 0$$

$$f'(x) = \int x^{-3/2} dx = -2x^{-1/2} + C$$

$$f'(x) = -\frac{2}{\sqrt{x}} + C$$

$$2 = -\frac{2}{\sqrt{4}} + C$$

$$2 = -\frac{2}{2} + C$$

$$2 = -1 + C$$

$$3 = C$$

$$f'(x) = -2x^{-1/2} + 3$$

$$f(x) = \int (-2x^{-1/2} + 3) dx$$

$$f(x) = -4x^{1/2} + 3x + C$$

$$0 = -4(0)^{1/2} + 3(0) + C$$

$$0 = C$$

$$\therefore f(x) = -4\sqrt{x} + 3x$$

12. An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by the differential equation

$$\frac{dh}{dt} = 1.5t + 5,$$

where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted, at $t = 0$. $h(0) = 12$

- a. Find the value of the differential equation above when $t = 3$. Using correct units of measure, explain what this value represents in the context of this problem.

$$\begin{aligned} \left. \frac{dh}{dt} \right|_{t=3} &= 1.5(3) + 5 \\ &= 4.5 + 5 \\ &= 9.5 \text{ cm/year} \end{aligned}$$

At 3 years, the trees are growing at a rate of 9.5 cm per year.

- b. Find an equation for $h(t)$, the height of the shrubs at any year t . Then, determine how tall the shrubs are when they are sold.

$$\begin{aligned} h(t) &= \int \frac{dh}{dt} dt \\ &= \int (1.5t + 5) dt \end{aligned}$$

$$h(t) = 0.75t^2 + 5t + C$$

$$\begin{aligned} h(0) &= 12 \\ 12 &= 0.75(0)^2 + 5(0) + C \\ 12 &= C \end{aligned}$$

$$h(t) = 0.75t^2 + 5t + 12$$

$$\begin{aligned} h(6) &= 0.75(6)^2 + 5(6) + 12 \\ &= 0.75(36) + 30 + 12 \\ &= 27 + 42 \end{aligned}$$

$$h(6) = 69 \text{ cm}$$

At 6 years, the height of the tree is 69 cm tall.

13. A particle moves along the x -axis at a velocity of $v(t) = \frac{1}{\sqrt{t}}$, for $t > 0$. At time $t = 1$, its position is 4.

a. What is the acceleration of the particle when $t = 9$?

$$p(1) = 4$$

$$a(t) = v'(t) = -\frac{1}{2} t^{-\frac{3}{2}}$$

$$a(9) = -\frac{1}{2(\sqrt{9})^3}$$

$$= -\frac{1}{2(3)^3}$$

$$= -\frac{1}{2(27)}$$

$$a(9) = -\frac{1}{54} \text{ linear unit/time}^2$$

b. What is the position of the particle when $t = 9$?

$$p(t) = \int v(t) dt = \int t^{\frac{1}{2}} dt$$

$$p(t) = 2t^{\frac{3}{2}} + C$$

$$\boxed{p(1) = 4}$$

$$4 = 2\sqrt{1} + C$$

$$4 = 2 + C$$

$$2 = C$$

$$p(t) = 2\sqrt{t} + 2$$

$$p(9) = 2\sqrt{9} + 2$$

$$= 2 \cdot 3 + 2$$

$$p(9) = 6 + 2$$

$$= 8 \text{ linear units.}$$