Notes 6.7 – Average Value of a Function

How have we found Average Velocity?

$$A.V. = \frac{p(a) - p(b)}{a - b}$$

How have we found Average Acceleration?

$$A.A. = \frac{v(a) - v(b)}{a - b}$$

If $p(t)$, $v(t)$, and $a(t)$ represent position, velocity and acceleration defined for any time t, write an equivalent expression for each of the following integrals based on the fundamental theorem of calculus.

$\frac{1}{b-a}\int_a^b a(t)dt =$	$\frac{1}{b-a} v(t)\Big\vert_a^b$	To what is this equivalent? A.A.
$\frac{1}{b-a}\int_a^b v(t)dt =$	$\frac{1}{b-a} p(t)\Big\vert_a^b$	To what is this equivalent? A.V.

The average value of a function, $f(x)$, on an interval $[a, b]$ is defined to be:

$$\frac{1}{b-a}\int_a^b f(x)dx$$

1. Find the average value of the function $f(x) = x^3\sqrt{sin^2 x}$ on the interval $1 \leq x \leq 3$. [Calculator]

$$A.\text{Value} = \frac{1}{3-1}\int_1^3 x^3\sqrt{\sin^2 x}\,dx$$

$$= \frac{1}{2}(11.696)$$

$$= 5.848$$

2. Find the average value of the function $f(x) = 2 - 4x$ on the interval $2 \leq x \leq 6$. [Noncalculator]

$$A.\text{value} = \frac{1}{6-2}\int_2^6 (2-4x)dx$$

$$= \frac{1}{4}\left[2x - 2x^2\right]\Big\vert_2^6$$

$$= \frac{1}{4}\left[(2\cdot 6 - 2(6)^2) - (2\cdot 2 - 2(2)^2)\right]$$

$$= \frac{1}{4}\left[(12 - 2\cdot 36) - (4 - 2\cdot 4)\right]$$

$$= \frac{1}{4}\left[(12-72) - (4-8)\right]$$

$$= \frac{1}{4}\left[-60 + 4\right]$$

$$= \frac{1}{4}(-56)$$

$$= -14$$

3. A ski resort uses a snow machine to control the snow level on a ski slope. Over a 24-hour period the volume of snow added to the slope per hour is modeled by the equation $S(t) = 24 - t\sin^2\left(\frac{t}{14}\right)$. The rate at which the snow melts is modeled by the equation $M(t) = 10 + 8\cos\left(\frac{t}{3}\right)$. Both $S(t)$ and $M(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 24$. At time $t = 0$, the slope holds 50 cubic yards of snow. $T(0)=50$

$\boxed{\text{CALC}}$

a. Compute the total volume of snow added to the mountain over the first 6-hour period.

$$\int_0^6 \left[24 - t\sin^2\left(\frac{t}{14}\right)\right] dt = 142.413 \text{ yd}^3$$

b. Find the value of $\int_0^6 M(t)dt$ and $\frac{1}{6}\int_0^6 M(t)dt$. Using correct units of measure, explain what each represents in the context of this problem.

$$\int_0^6 M(t)dt = 81.823 \text{ yds}^3$$

$$\frac{1}{6}\int_0^6 M(t)dt = 13.637 \text{ yds}^3/\text{hr}$$

$\int_0^6 M(t)dt$ represents the total amount of snow melt from $t=0$ to $t=6$ hours.

$\frac{1}{6}\int_0^6 M(t)dt$ represents the average amount of snow that melts per hour from $t=0$ to $t=6$ hours

c. Is the volume of snow increasing or decreasing at time $t = 4$? Justify your answer.

Total Snow $= T(t) = T(0) + \int_0^t S(t)dt - \int_0^t M(t)dt$

$T(t) = 50 + \int_0^t S(t)dt - \int_0^t M(t)dt$

$T'(t) = S(t) - M(t)$

$T'(4) = S(4) - M(4)$

$= 11.800$

Since $T'(4) > 0$, the volume of snow is increasing at $t=4$.

d. How much snow is on the slope after 5 hours? Show your work.

$T(5) = 50 + \int_0^5 S(t)dt - \int_0^5 M(t)dt$

$= 95.335 \text{ yds}^3$

e. Suppose the snow machine is turned off at time $t = 10$. Write, but do not solve, an equation that could be solved to find the time $t = K$ when the snow would all be melted.

$$T(10) - \int_{10}^{K} M(t)dt = 0$$