Notes 7.1 – The Second Fundamental Theorem of Calculus
Functions Defined by Integrals

2nd Fun'l Th'm of Calculus

Assume a is any constant

If $F(x) = \int_a^{g(x)} f(t)\,dt$,

then $F'(x) = f(g(x)) \cdot g'(x)$

Find the derivative of each of the following functions.

1. $F(x) = \int_{-2}^{2x} \sqrt{2-t^2}\,dt$

$F'(x) = \sqrt{2-(2x)^2} \cdot (2x)'$

$F'(x) = 2\sqrt{2-4x^2}$

2. $G(x) = \int_{x^2}^{-3} e^{\cos t}\,dt$

$G(x) = -\int_{-3}^{x^2} e^{\cos t}\,dt$

$G'(x) = -e^{\cos x^2} \cdot (x^2)'$

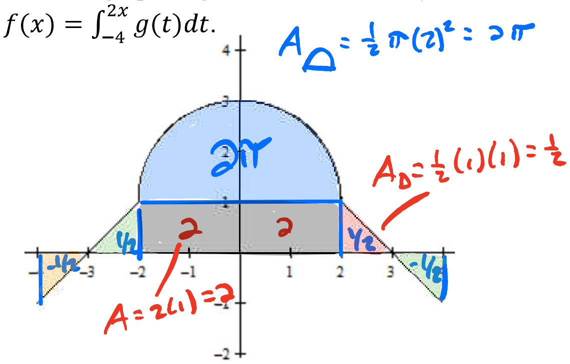
$G'(x) = -2x\,e^{\cos x^2}$

3. $H(x) = \int_0^{\cos x} t^2\,dt$

$H'(x) = (\cos x)^2 \cdot (\cos x)'$

$H'(x) = -\sin x \cdot \cos^2 x$

Pictured is the graph of $g(t)$ and the function $f(x)$ is defined to be $f(x) = \int_{-4}^{2x} g(t)\,dt$.



4. Find the value of $f(0)$.

$f(0) = \int_{-4}^{2(0)} g(t)\,dt = 2 + \pi$

5. Find the value of $f(2)$.

$f(2) = \int_{-4}^{2(2)} g(t)\,dt = 4 + 2\pi$

6. Find the value of $f'(1)$.

$f'(x) = g(2x) \cdot (2x)'$

$f'(x) = 2 \cdot g(2x)$

$f'(1) = 2 \cdot g(2 \cdot 1)$

$= 2 \cdot 1$

$f'(1) = 2$

7. Find the value of $f'(-2)$.

$f'(x) = 2 \cdot g(2x)$

$f'(-2) = 2 \cdot g(2(-2))$

$f'(-2) = 2g(-4)$

$= 2 \cdot (-1)$

$f'(-2) = -2$

8. Find the value of $f''(2)$.

$f''(x) = 2g'(2x) \cdot (2x)'$

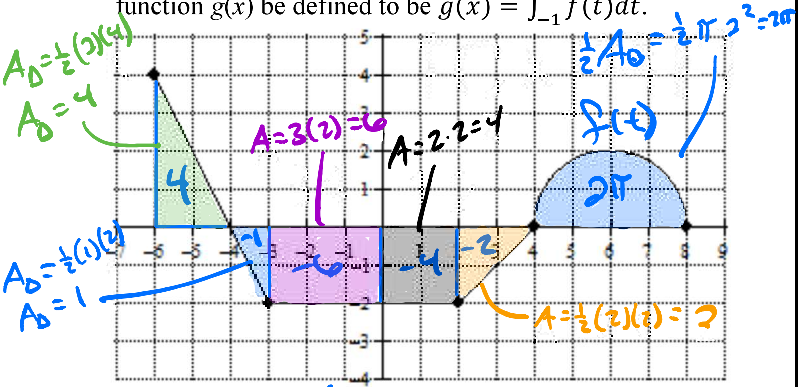
$f''(x) = 4g'(2x)$

$f''(2) = 4g'(2 \cdot 2)$

$= 4 \cdot g'(4)$

$f''(2) =$ undefined b/c $g'(4)$ does not exist

Given is the graph of $f(t)$ which consists of three line segments and one semicircle. Additionally, let the function $g(x)$ be defined to be $g(x) = \int_{-1}^{x} f(t)\,dt$.



1. Find $g(-6)$.

$$= \int_{-1}^{-6} f(t)\,dt$$

$$= -\int_{-6}^{-1} f(t)\,dt$$

$$= -(4 - 1 - 4)$$

$$= -(-1)$$

$$g(-6) = 1$$

2. Find $g(6)$.

$$g(6) = \int_{-1}^{6} f(t)\,dt$$

$$= -6 + -2 + \pi$$

$$= -8 + \pi$$

3. Find $g'(6)$.

$$g'(x) = f(x) \cdot x'$$

$$g'(x) = f(x)$$

$$g'(6) = f(6)$$

$$g'(6) = 2$$

4. Find $g'(2)$.

$$g'(x) = f(x)$$

$$g'(2) = f(2)$$

$$g'(2) = -2$$

5. Find $g''(2)$. Give a reason for your answer.

$$g''(x) = f'(x)$$

$$g''(2) = f'(2) = \text{undefined}$$

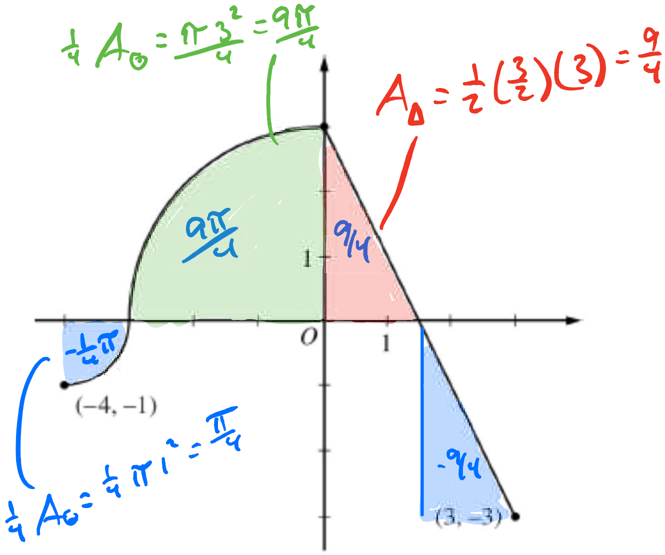
because $f(t)$ has a cusp at $t = 2$.

6. Find $g''(-4)$. Give a reason for your answer.

$$g''(x) = f'(x)$$

$$g''(-4) = f'(-4) = -2$$

because $f(t)$ has a slope of -2 at $t = -4$.



Graph of f

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph consists of two quarter circles and one line segment, as show in the figure above.

Let $g(x) = \frac{1}{2}x^2 + \int_0^x f(t)dt$.

9. Find the value of $g(3)$.

$g(3) = \frac{1}{2}(3)^2 + \int_0^3 f(t)\,dt$

$= \frac{9}{2} + \frac{9}{4} + (-\frac{9}{4})$

$= 9/2$

10. Find the value of $g'(3)$.

$g'(x) = x + f(x) \cdot x'$

$g'(x) = x + f(x)$

$g'(3) = 3 + f(3)$

$= 3 + (-3)$

$g'(3) = 0$

11. Find the value of $g(-4)$.

$g(-4) = \frac{1}{2}(-4)^2 + \int_0^{-4} f(t)dt$

$= 8 - (-\frac{\pi}{4} + \frac{9\pi}{4})$

$= 8 - (\frac{8\pi}{4})$

$= 8 - 2\pi$

12. Find the value of $g''(2)$.

$g'(x) = x + f(x)$

$g''(x) = 1 + f'(x)$

$g''(2) = 1 + f'(2)$

$= 1 + (-2)$

$g''(2) = -1$