Notes 7.4 – Slope Fields

Graphical Representations of Solutions to Differential Equations

A **slope field** is a pictorial representation of all of the possible solutions to a given differential equation.

Remember that a differential equation is the first derivative of a function, $f'(x)$ or $\frac{dy}{dx}$. Thus, the solution to a differential equation is the function, $f(x)$ or y.

There is an infinite number of solutions to the differential equation $\frac{dy}{dx} = x - 1$. Show your work and explain why.

$\int dy = \int (x-1)\,dx$

$y = \frac{1}{2}x^2 - x + C$

$f(x) = \frac{1}{2}x^2 - x + C$ is a general solution of $\frac{dy}{dx} = x+3$ and it represents all of the quadratic functions that could be $f(x)$ for $\frac{dy}{dx}$.

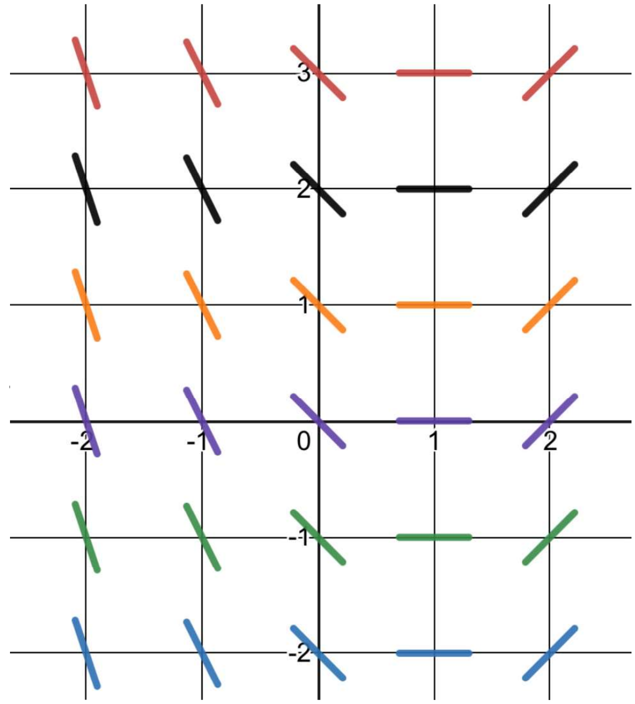
For the AP Exam, you are expected to be able to do the following four things with slope fields:

1. Given a differential equation $\frac{dy}{dx}$, sketch a slope field.
2. Given a slope field, draw a particular solution
3. match a slope field to a differential equation.
4. match a slope field to a solution of a differential

Given the differential equation below, compute the slope for each point
Indicated on the grid to the right.

Then, make a small mark that approximates the slope through the point.

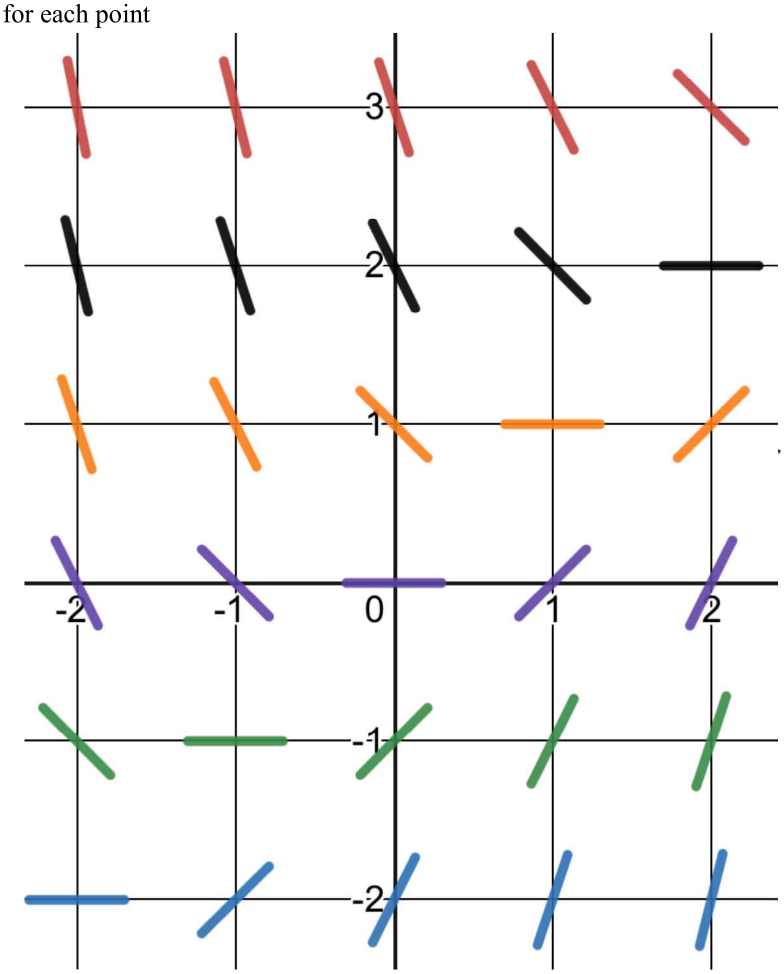
$$\frac{dy}{dx} = x - 1$$



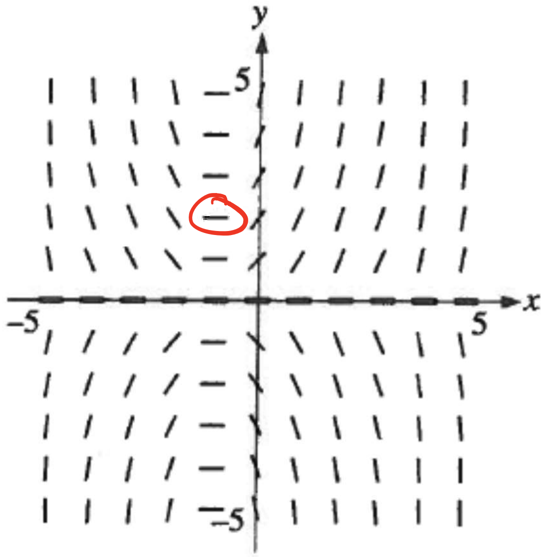
Notice how the segments drawn on the grid above would form parabolas if they were connected.
Given the differential equation below, compute the slope for each point
Indicated on the grid to the right.

Then, make a small mark that approximates the slope through the point.

$$\frac{dy}{dx} = x - y$$



Shown below is a slope field for which of the following differential equations? Explain your reasoning for each of the choices below.



X (A) $\frac{dy}{dx} = xy$ = -2 at $(-1, 2)$

X (B) $\frac{dy}{dx} = xy - y$ = -4 at $(-1, 2)$

(C) $\frac{dy}{dx} = xy + y$ = 0 at $(-1, 2)$

X (D) $\frac{dy}{dx} = xy + x$ = 2 at $(2, 0)$

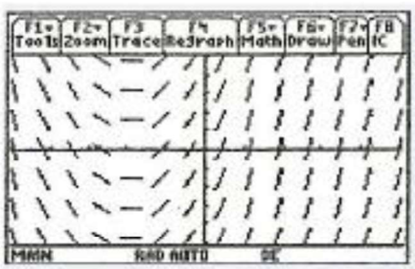
X (E) $\frac{dy}{dx} = (x+1)^3$ = 8 at $(2, 0)$

#3 Match a slope field to a differential equation.

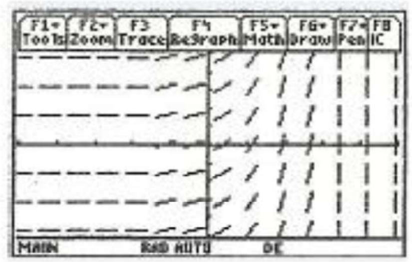
Since the slope field represents all of the particular solutions to a differential equation, and the solution represents the ANTIDERIVATIVE of a differential equation, then the slope field should take the shape of the antiderivative of *dy/dx*.

Match the slope fields to the differential equations on the next page.

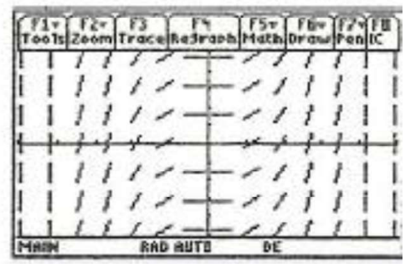
A.



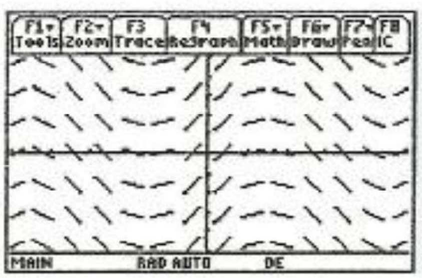
B.



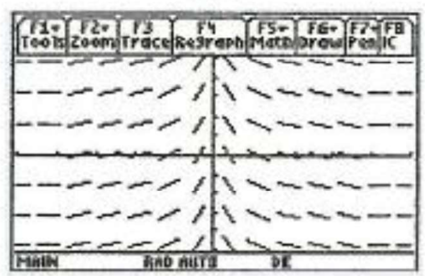
C.



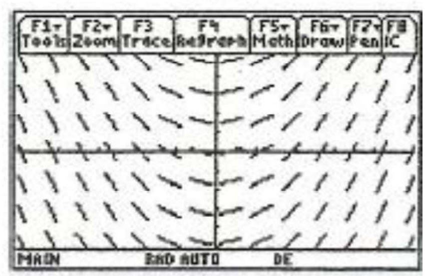
D.



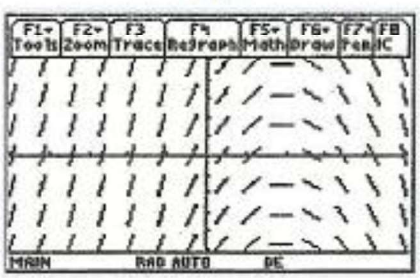
E.



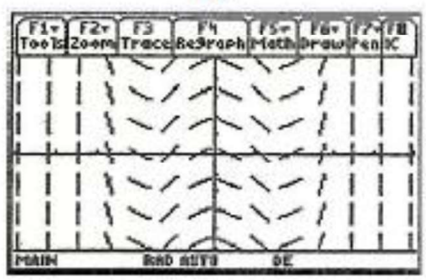
F.



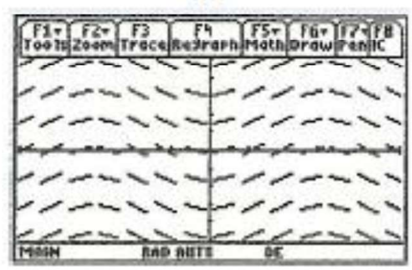
G.



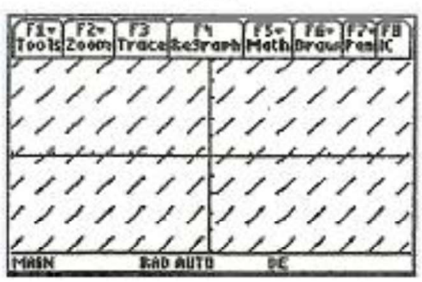
H.



I.



J.



Separate the variables and find the general solution to each differential equation below to determine what the slope field should look like for each. Then, match to the graphs of slope fields on the previous page.

I	1. $\frac{dy}{dx} = \sin x$ $\int dy = \int \sin x \, dx$ $y = -\cos x + C$ D or I	A	2. $\frac{dy}{dx} = 2x + 4$ $\int dy = \int (2x+4)\,dx$ $y = x^2 + 4x + C$ F or A
B	3. $\frac{dy}{dx} = e^x$ $\int dy = \int e^x \, dx$ $y = e^x + C$	J	4. $\frac{dy}{dx} = 2$ $\int dy = \int 2 \, dx$ $y = 2x + C$
H	5. $\frac{dy}{dx} = x^3 - 3x$ $\int dy = \int (x^3 - 3x)\,dx$ $y = \frac{1}{4}x^4 - \frac{3}{2}x^2 + C$	D	6. $\frac{dy}{dx} = 2\cos x$ $\int dy = \int 2\cos x \, dx$ $y = 2\sin x + C$
G	7. $\frac{dy}{dx} = 4 - 2x$ $\int dy = \int (4 - 2x)\,dx$ $y = 4x - x^2 + C$	F	8. $\frac{dy}{dx} = x$ $y = \frac{1}{2}x^2 + C$
C	9. $\frac{dy}{dx} = x^2$ $\int dy = \int x^2 \, dx$ $y = \frac{1}{3}x^3 + C$	E	10. $\frac{dy}{dx} = -\frac{1}{x}$ $y = -\ln\lvert x \rvert + C$