

Notes 1.1 – Understanding the Limit

A Numerical and Graphical Approach

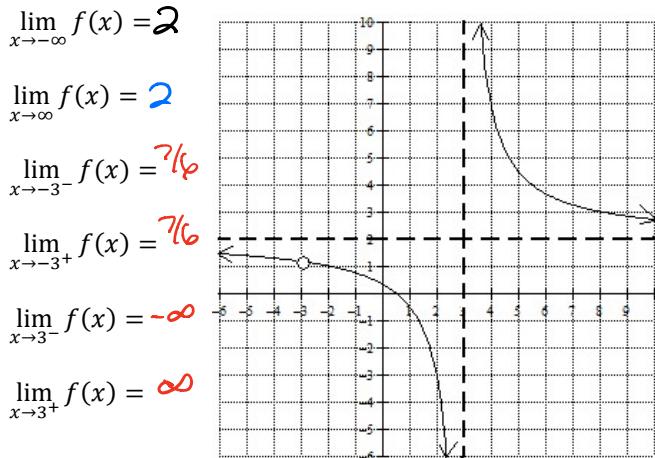
The equation of the function graphed to the right is $f(x) = \frac{2x^2+5x-3}{x^2-9}$. The coordinates of the hole in the graph are $(-3, \frac{7}{6})$.

$$\begin{array}{l}
 \text{HA @ } y = 2 \\
 \text{VA @ } x = 3 \\
 \text{zero } x = y_2 \\
 y\text{-int} = y_3
 \end{array}
 \quad \left| \begin{array}{l}
 f(x) = \frac{2x^2+5x-3}{x^2-9} \\
 = \frac{(2x-1)(x+3)}{(x-3)(x+3)} \\
 = \frac{2x-1}{x-3}
 \end{array} \right.$$

Hole @ $x = -3$ $(-3, \frac{7}{6})$

$x+3=0$ $x = -3$	$f(x) = \frac{2x-1}{x-3}$ $= \frac{+7}{+6}$
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$$\lim_{x \rightarrow -3} f(x) \neq f(-3)$$



Based on what you have just seen, how might you informally define what the value of a limit represents in terms of the graph?

$\lim_{x \rightarrow a} f(x)$ is the y -value the graph approaches as $x \rightarrow a$

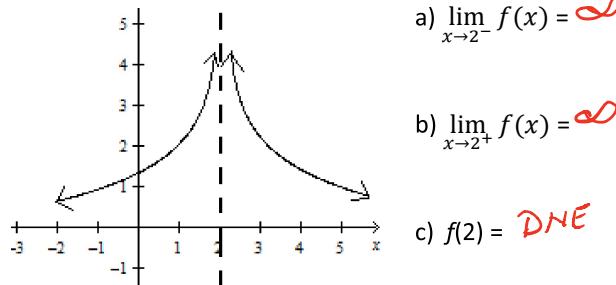
Limit Existence Theorem

$\lim_{x \rightarrow a} f(x)$ exists if and only if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = b$, where b is any real number

Limits That Do Not Exist

Find each of the following from the graph.

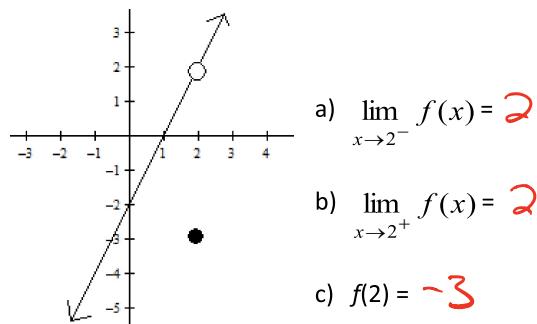
Example #1



d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

No, $\lim_{x \rightarrow 2} f(x) = \infty$, which is not a number

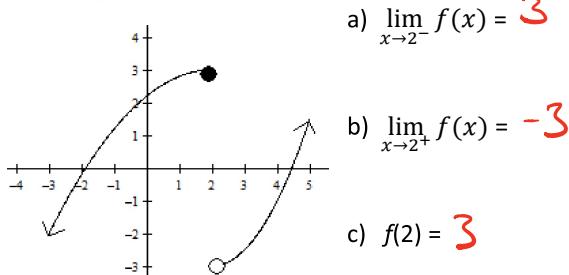
Example #2



d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

Yes, $\lim_{x \rightarrow 2} f(x)$ exists because $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = 2$

Example #3

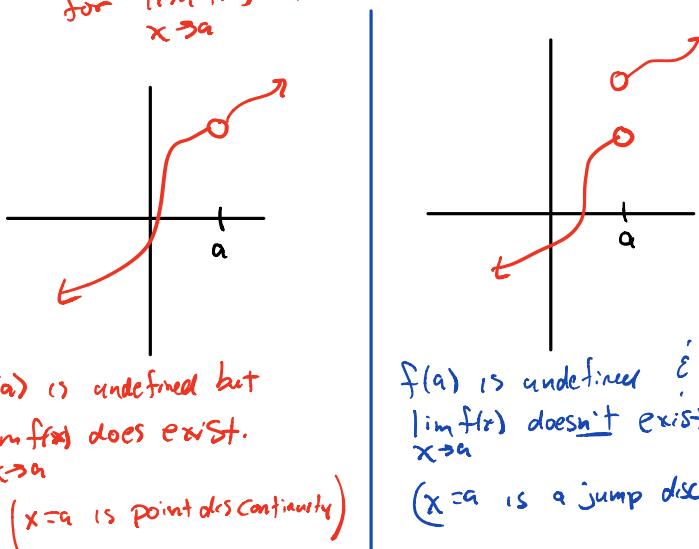


d) Does $\lim_{x \rightarrow 2} f(x)$ exist or not? Why or why not?

No, $\lim_{x \rightarrow 2} f(x)$ does not exist because $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$

Based on what you have seen so far, does $f(a)$ have to be defined in order for the $\lim_{x \rightarrow a} f(x)$ to exist? Draw and explain two different graphs to justify your reasoning. In both graphs, $f(a)$ should be undefined but in one graph, the limit should exist while in the second one, it should not exist.

No, $f(a)$ does not need to be defined for $\lim_{x \rightarrow a} f(x)$ to exist.



$f(a)$ is undefined but $\lim_{x \rightarrow a} f(x)$ does exist.
 $(x=a$ is point discontinuity)

$f(a)$ is undefined & $\lim_{x \rightarrow a} f(x)$ doesn't exist
 $(x=a$ is a jump disc.)

Below is a table of values of an exponential function. Use the table to find the limits that follow.

x	-9	-5	-3	-1	1	3	9
$f(x)$	513	33	9	3	1.5	1.125	1.002

$$\begin{array}{ll} a) \lim_{x \rightarrow \infty} f(x) = \infty & b) \lim_{x \rightarrow -3} f(x) = 9 \\ c) \lim_{x \rightarrow 1} f(x) = 1.5 & d) \lim_{x \rightarrow \infty} f(x) = 1 \end{array}$$

Below is a table of values of a rational function. Use the table to find the limits that follow.

VA

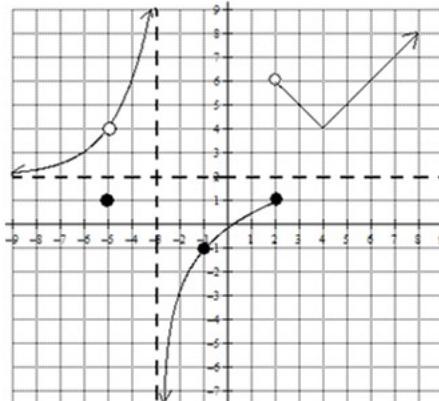
Hole

x	-1000	-1.001	-1	-0.999	0	1.999	2	2.001	1000
$F(x)$	1.002	2001	Undefined	-1999	-1	0.333	Undefined	0.334	0.998

$$\begin{array}{lll} a) \lim_{x \rightarrow -\infty} f(x) = 1 & b) \lim_{x \rightarrow -1^-} f(x) = \infty & c) \lim_{x \rightarrow -1^+} f(x) = -\infty \\ d) \lim_{x \rightarrow 2} f(x) = \frac{1}{3} & e) \lim_{x \rightarrow \infty} f(x) = 1 & f) \lim_{x \rightarrow 1} f(x) = \text{dne} \end{array}$$

A Graphical Analysis of Limits

Using the graphs of $f(x)$ or $g(x)$ find the value of each of the following limits. If a limit does not exist, state why



$$\begin{array}{lll} a) \lim_{x \rightarrow -3^-} f(x) = \infty & b) \lim_{x \rightarrow -5} f(x) = 4 & c) \lim_{x \rightarrow -1} f(x) = -1 \end{array}$$

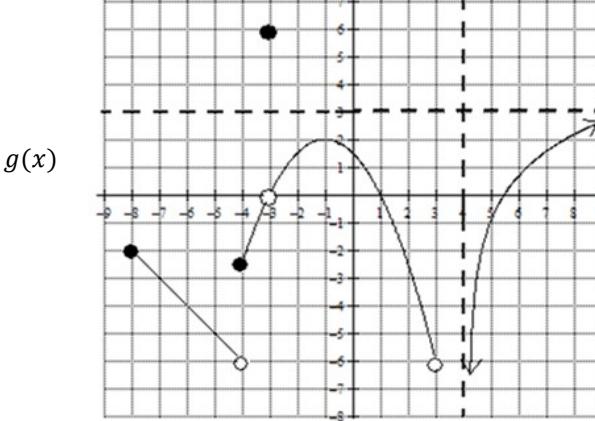
DNE, $\infty \neq \#$

$$\begin{array}{lll} c) \lim_{x \rightarrow -3} f(x) = \text{dne} & d) \lim_{x \rightarrow 2^-} f(x) = 1 & e) \lim_{x \rightarrow 2^+} f(x) = 6 \end{array}$$

$\lim_{x \rightarrow -3^-} f(x) \neq \lim_{x \rightarrow -3^+} f(x)$

$$\begin{array}{lll} f) \lim_{x \rightarrow 2} f(x) = \text{DNE} & g) \lim_{x \rightarrow \infty} f(x) = 2 & h) \lim_{x \rightarrow -\infty} f(x) = \infty \end{array}$$

DNE, $\infty \neq \#$



$$\begin{array}{lll} a) \lim_{x \rightarrow -3^-} g(x) = 0 & b) \lim_{x \rightarrow -6} g(x) = -4 & c) \lim_{x \rightarrow 1^+} g(x) = 2 \end{array}$$

$$\begin{array}{lll} d) \lim_{x \rightarrow -3^+} g(x) = 0 & e) \lim_{x \rightarrow 4^-} g(x) = \text{DNE} & f) \lim_{x \rightarrow 4^+} g(x) = -\infty \end{array}$$

$$\begin{array}{lll} g) \lim_{x \rightarrow 4} g(x) = \text{DNE} & h) \lim_{x \rightarrow 4^+} g(x) = -2 & i) \lim_{x \rightarrow 4^-} g(x) = -6 \end{array}$$

$\lim_{x \rightarrow 4^-} g(x) \neq \lim_{x \rightarrow 4^+} g(x)$