Notes 1.3 – Properties for Limits

Graphical and Analytical Connections

Suppose $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Find each of the following limits in terms of L and M.

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

   $= L + M$

2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

   $= L - M$

3. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$

4. $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

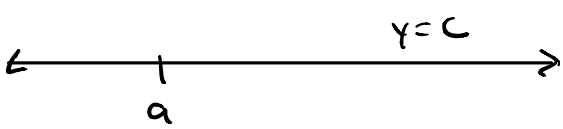
   $= L \cdot M$

5. $\lim_{x\to a}[c\cdot f(x)] = c \cdot \lim_{x\to a} f(x)$

   $= c \cdot L$

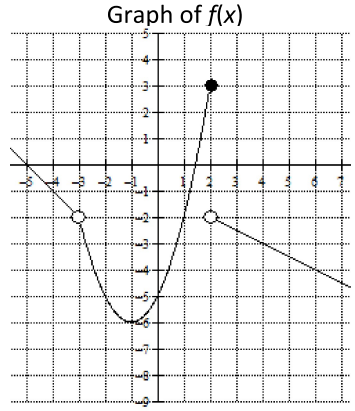
6. $\lim_{x\to a}[f(x)]^p = \left[\lim_{x\to a} f(x)\right]^p = L^p$

7. $\lim_{x\to a} c = c$

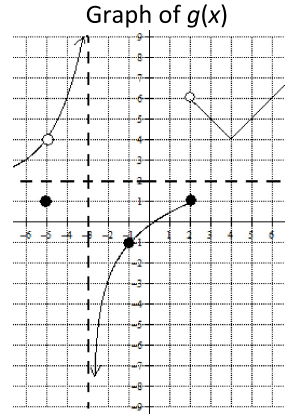


Find each limit. If a limit does not exist, state why.

Graph of $f(x)$



Graph of $g(x)$



1) $\lim_{x\to -1}[2f(x)-3g(x)]$

$= 2\lim_{x\to -1} f(x) - 3\lim_{x\to -1} g(x)$

$= 2(-6) - 3(-1)$

$= -12 + 3$

$= -9$

2) $\lim_{x\to -3}[f(x)-g(x)]$

$= \lim_{x\to -3} f(x) - \lim_{x\to -3} g(x)$

$= (-2) - (\text{DNE})$

$= \text{DNE}$ ($g(x)$ has VA at $x=-3$)

3) $\lim_{x\to 6} \frac{-2f(x)}{g(x)}$

$= \frac{-2\lim_{x\to 6} f(x)}{\lim_{x\to 6} g(x)} = \frac{-2(-4)}{6}$

$= \frac{8}{6}$

$= 4/3$

4) $\lim_{x\to -2}[f(x)]^2$

$= \left[\lim_{x\to -2} f(x)\right]^2$

$= [-5]^2$

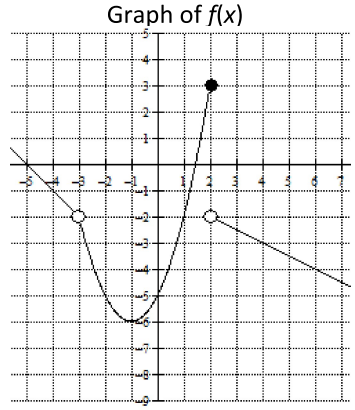
$= 25$

5) $\lim_{x\to 2^+} \sqrt{2g(x)} = \sqrt{2\cdot(6)}$

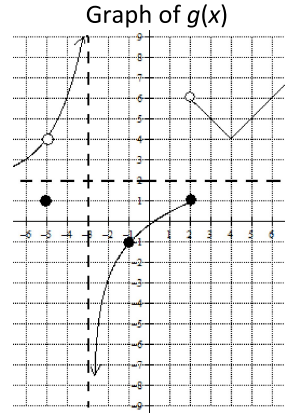
$= \sqrt{12}$

$= 2\sqrt{3}$

Graph of $f(x)$



Graph of $g(x)$



6) If $g(x) = 3x + 2$, find $\lim_{x\to 0} \frac{2g(x)+g(-2)}{x}$.

$$= \lim_{x\to 0} \frac{2(3x+2) + (3(-2)+2)}{x}$$

$$= \lim_{x\to 0} \frac{6x+4-6+2}{x}$$

$$= \lim_{x\to 0} \frac{6\cancel{x}}{\cancel{x}}$$

$$= \lim_{x\to 0} 6$$

$$= 6$$

7) If $f(x) = 2x^2 - 3x + 4$, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[2(x+h)^2 - 3(x+h) + 4] - [2x^2 - 3x + 4]}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2hx+h^2) - 3x - 3h + 4 - 2x^2 + 3x - 4}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{3x} - 3h \cancel{+4} - \cancel{2x^2} \cancel{+3x} \cancel{-4}}{h}$$

$$= \lim_{h\to 0} \frac{4hx + 2h^2 - 3h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4x + 2h - 3)$$

$$= 4x + 2(0) - 3$$

$$= 4x - 3$$