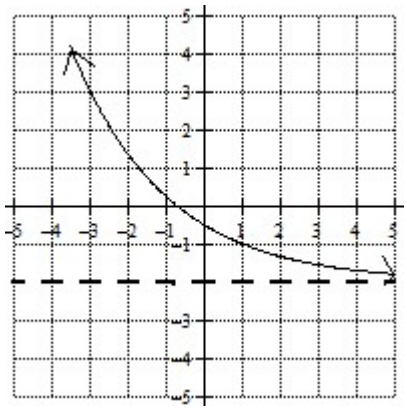
Notes 1.4 – Limits of Exponential Functions
Graphical and Analytical Connections

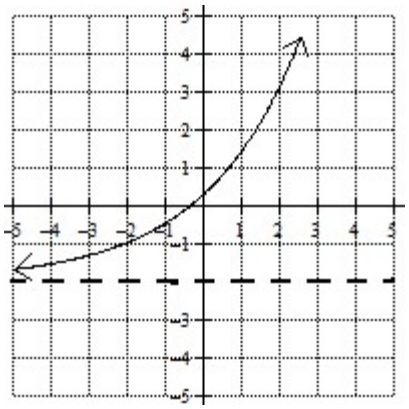
Consider the four exponential functions graphed below. Find the indicated limits for each function based on the graph.

$f(x) = \left(\frac{2}{3}\right)^{x-1} - 2 = \left(\frac{3}{2}\right)^{-(x-1)} - 2$



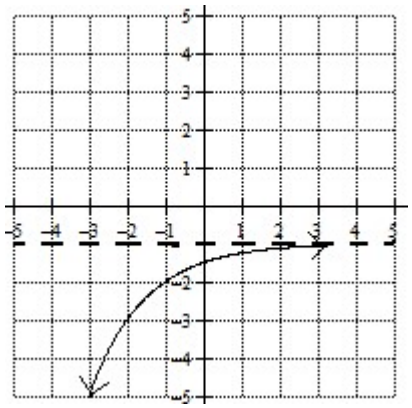
$\lim_{x\to-\infty} f(x) = \infty$ DNE $\quad \lim_{x\to\infty} f(x) = -2$

$f(x) = \left(\frac{3}{2}\right)^{x+2} - 2$



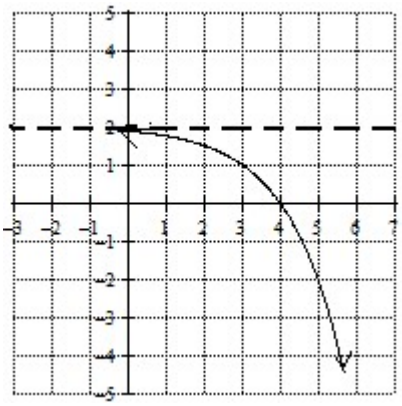
$\lim_{x\to-\infty} f(x) = -2 \quad \lim_{x\to\infty} f(x) = \infty$ dne

$f(x) = -\left(\frac{1}{2}\right)^{x+1} - 1 = -(2)^{-(x+1)} - 1$



$\lim_{x\to-\infty} f(x) = -\infty$ dne $\quad \lim_{x\to\infty} f(x) = -1$

$f(x) = -\left(\frac{1}{2}\right)^{-x+3} + 2 = -(2)^{x-3} + 2$



$\lim_{x\to-\infty} f(x) = 2 \quad \lim_{x\to\infty} f(x) = -\infty$ dne

In order to determine a limit as x approaches $-\infty$ or ∞ for an exponential function, you have to determine what the graph will look like. Based on what we have seen above, what are the three possible results of such a limit for an exponential function?

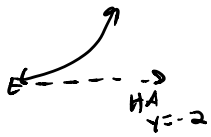
$-\infty$	∞	The y-value of HA

By studying the graphs above, remind yourself of the parent graphs of exponential functions and transformations.

Determine the limits of each of the following exponential functions.

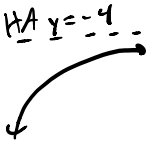
1. $\lim_{x\to\infty}\left[\left(\frac{2}{3}\right)^{-x-1}-2\right] = \lim_{x\to\infty}\left[\left(\frac{3}{2}\right)^{x+1}-2\right]$

$= \infty$, dne



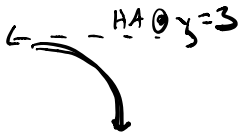
2. $\lim_{x\to-\infty}[-(0.4)^x-4] = \lim_{x\to-\infty}\left[-\left(\frac{10}{4}\right)^{-x}-4\right]$

$= -\infty$, dne

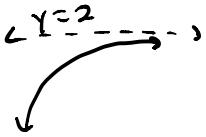


3. $\lim_{x\to\infty}\left[-\left(\frac{2}{3}\right)^{-x+2}+3\right]$

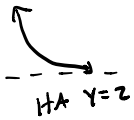
$= \lim_{x\to\infty}\left[-\left(\frac{3}{2}\right)^{x-2}+3\right] = -\infty$



4. $\lim_{x\to\infty}[-(2)^{-x-1}+2] = 2$

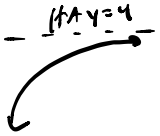


5. $\lim_{x\to-\infty}[e^{-x-1}+2] = \infty$



6. $\lim_{x\to\infty}[-(0.4)^x-4]$

$= \lim_{x\to\infty}\left[-\left(\frac{10}{4}\right)^{-x}-4\right] = 4$



7. $\lim_{x\to3}(e^{2-x}+2) = e^{2-3}+2$

$= e^{-1}+2$

$= \frac{1}{e}+2$

8. $\lim_{x\to-2}\left[\left(\frac{1}{2}\right)^{-x-3}+3\right] = \left(\frac{1}{2}\right)^{-(-2)-3}+3$

$= \left(\frac{1}{2}\right)^{-1}+3$

$= 2^1+3$

$= 5$