

**Notes 1.6 – Limit-Based Continuity***Graphical and Analytical Approaches*

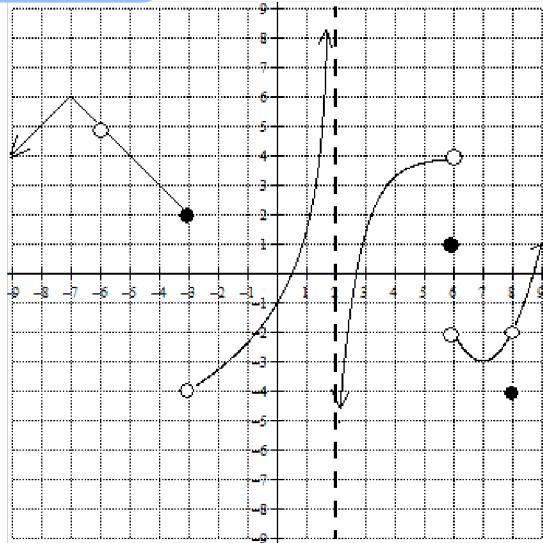
For the function graphed below, fill in the table with the given information. After filling in the table, write three pieces of information that must be true in order for a function,  $G(x)$ , to be continuous at  $x = a$ .

**Three part definition of continuity**

1.  $G(a)$  is defined

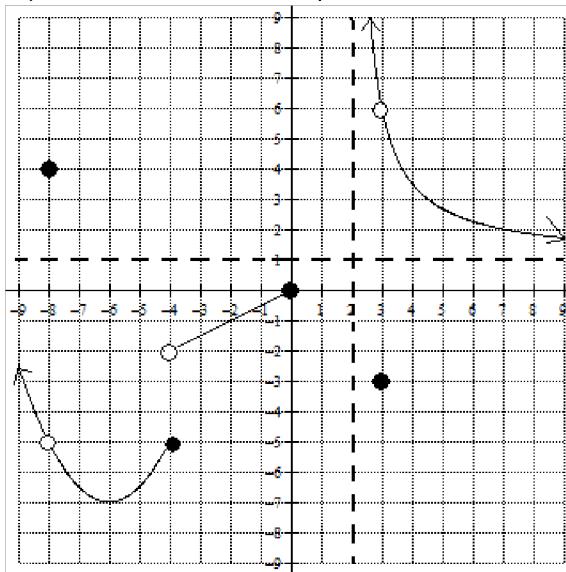
2.  $\lim_{x \rightarrow a} G(x)$  exists

3.  $\lim_{x \rightarrow a} G(x) = G(a)$



$x = a$	Is the function defined? If so, what is its value?	What is the value of $\lim_{x \rightarrow a^-} G(x)$ ?	What is the value of $\lim_{x \rightarrow a^+} G(x)$ ?	What is $\lim_{x \rightarrow a} G(x)$ ?	Is $G(x)$ continuous at $x = a$ ?
$x = -6$	$G(-6)$ is undefined	5	5	5	NO
$x = -3$	$G(-3) = 2$	2	-4	DNE	NO
$x = 0$	$G(0) = -1$	-1	-1	-1	Yes
$x = 2$	$G(2)$ is undefined	$\infty$	$-\infty$	DNE	NO
$x = 6$	$G(6) = 1$	4	-2	DNE	NO
$x = 8$	$G(8) = -4$	-2	-2	-2	NO

The graph of the function,  $G(x)$ , pictured below has several  $x$ -values at which the function is not continuous. For each of the following  $x$ -values, use the three part definition of continuity to determine if the function is continuous or not.



1.  $x = -8$

$$\text{I. } G(-8) = 4$$

$$\therefore G(x) \text{ is defined}$$

$$\text{II. } \lim_{x \rightarrow -8^-} G(x) = \lim_{x \rightarrow -8^+} G(x) = -5$$

$$\therefore \lim_{x \rightarrow -8} G(x) = -5$$

$$\therefore \lim_{x \rightarrow -8} G(x) \text{ exists}$$

$$\text{III. } G(-8) \neq \lim_{x \rightarrow -8} G(x)$$

$$\therefore G(x) \text{ is not continuous at } x = -8$$

2.  $x = -6$

$$\text{I. } G(-6) = -7$$

$$\therefore G(x) \text{ is defined}$$

$$\text{II. } \lim_{x \rightarrow -6^-} G(x) = \lim_{x \rightarrow -6^+} G(x) = -7$$

$$\therefore \lim_{x \rightarrow -6^-} G(x) = -7$$

$$\therefore \lim_{x \rightarrow -6} G(x) \text{ exists}$$

$$\text{III. } G(-6) = \lim_{x \rightarrow -6} G(x)$$

$$\therefore G(x) \text{ is continuous at } x = -6$$

3.  $x = -4$

$$\text{I. } G(-4) = -5$$

$$\therefore G(x) \text{ is defined}$$

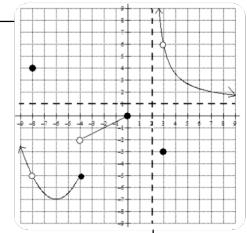
$$\text{II. } \lim_{x \rightarrow -4^-} G(x) = -5$$

$$\lim_{x \rightarrow -4^+} G(x) = -2$$

$$\therefore \lim_{x \rightarrow -4^-} G(x) \neq \lim_{x \rightarrow -4^+} G(x)$$

$$\therefore \lim_{x \rightarrow -4} G(x) \text{ does not exist}$$

$$\therefore G(x) \text{ is not continuous at } x = -4.$$



Use the three part definition of continuity to determine if the given functions are continuous at the indicated values of  $x$ .

Function	Domain
$f(x) = \begin{cases} -2\sqrt{x+6}, & x < -2 \\ 3x+2, & x = -2 \\ e^x + \cos(\pi x), & x > -2 \end{cases}$	at $x = -2$

$$\text{I. } f(-2) = 3(-2) + 2 = -6 + 2 = -4$$

$$\therefore f(-2) \text{ is defined}$$

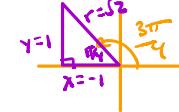
$$\text{II. } \lim_{x \rightarrow -2^-} (-2\sqrt{x+6}) = -2\sqrt{-2+6} = -2\sqrt{4} = -2 \cdot 2 = -4$$

$$\lim_{x \rightarrow -2^+} (e^x + \cos(\pi x)) = e^{-2} + \cos(\pi(-2)) = \frac{1}{e^2} + 1$$

$$\therefore \lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$$

$$\therefore \lim_{x \rightarrow -2} f(x) \text{ DNE}$$

$\therefore f(x)$  is not continuous at  $x = -2$



$$g(x) = \begin{cases} e^x \cos x, & x < \pi \\ e^x \tan\left(\frac{3x}{4}\right), & x \geq \pi \end{cases} \text{ at } x = \pi$$

$$\text{I. } g(\pi) = e^\pi \cdot \tan\left(\frac{3\pi}{4}\right) = e^\pi(-1) = -e^\pi$$

$$\text{II. } \lim_{x \rightarrow \pi^-} (e^x \cos x) = e^\pi \cos(\pi) = -e^\pi$$

$$\lim_{x \rightarrow \pi^+} (e^x \tan\left(\frac{3x}{4}\right)) = e^\pi \cdot \tan\left(\frac{3\pi}{4}\right) = e^\pi(-1) = -e^\pi$$

$$\text{Since } \lim_{x \rightarrow \pi^-} g(x) = \lim_{x \rightarrow \pi^+} g(x) = -e^\pi$$

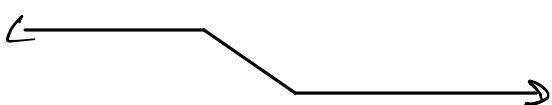
$$\therefore \lim_{x \rightarrow \pi} g(x) \text{ exists}$$

$$\text{III. } \lim_{x \rightarrow \pi} g(x) = g(\pi) = -e^\pi$$

$\therefore g(x)$  is continuous at  $x = \pi$

6. Consider the function,  $f(x)$ , to answer the following questions.

$$f(x) = \begin{cases} 2, & x \leq -1 \\ mx + k, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$$



a. What two limits must equal in order for  $f(x)$  to be continuous at  $x = -1$ ?

$$\lim_{x \rightarrow -1^-} 2 = \lim_{x \rightarrow -1^+} (mx + k)$$

$$\therefore 2 = m(-1) + k$$

b. What two limits must equal in order for  $f(x)$  to be continuous at  $x = 3$ ?

$$\lim_{x \rightarrow 3^-} (mx + k) = \lim_{x \rightarrow 3^+} (-2)$$

$$\therefore m(3) + k = -2$$

c. Determine the values of  $m$  and  $k$  so that the function is continuous everywhere.

$$\begin{aligned} 2 &= -m + k & \xrightarrow{\cdot 3} & 6 = -3m + 3k \\ -2 &= 3m + k & \xrightarrow{\quad} & -2 = 3m + k \\ 4 &= 4k & \xrightarrow{\quad} & 1 = k \end{aligned}$$

$$2 = -m + (1)$$

$$1 = -m$$

$$\boxed{-1 = m}$$

7. Consider the function,  $g(x)$ , to answer the following questions.

$$g(x) = \begin{cases} kx^2 + m, & x < -2 \\ 4x + 1, & -2 \leq x \leq 3 \\ kx - m, & x > 3 \end{cases}$$

- a. What two limits must equal in order for  $g(x)$  to be continuous at  $x = -2$ ?

$$\lim_{x \rightarrow -2^-} (kx^2 + m) = \lim_{x \rightarrow -2^+} (4x + 1)$$

$$k(-2)^2 + m = 4(-2) + 1$$

$$4k + m = -8 + 1$$

$$4k + m = -7$$

- b. What two limits must equal in order for  $g(x)$  to be continuous at  $x = 3$ ?

$$\lim_{x \rightarrow 3^-} (4x + 1) = \lim_{x \rightarrow 3^+} (kx - m)$$

$$4(3) + 1 = k(3) - m$$

$$13 = 3k - m$$

ons.

$$g(x) = \begin{cases} kx^2 + m, & x < -2 \\ 4x + 1, & -2 \leq x \leq 3 \\ kx - m, & x > 3 \end{cases}$$

- c. Determine the values of  $m$  and  $k$  so that the function is continuous everywhere.

$$4k + m = -7 \quad \rightarrow 4\left(\frac{6}{7}\right) + m = -7$$

$$3k - m = 13 \quad \frac{24}{7} + m = \frac{-49}{7}$$


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$$7k = 6 \quad m = \frac{-73}{7}$$

$$k = \frac{6}{7}$$