Notes 1.7 – Intermediate Value Theorem

As we study calculus, we will study several different theorems. The first theorem of investigation is the Intermediate Value Theorem.

Intermediate Value Theorem

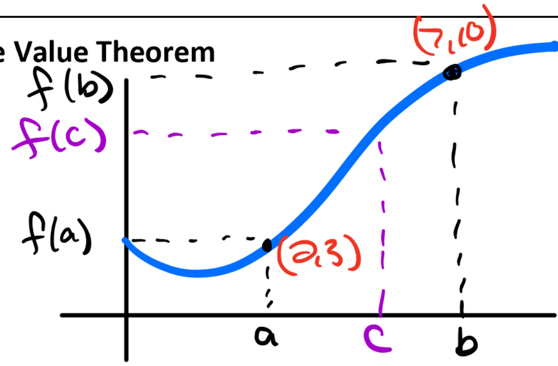
[2,7] 2 < 8 < 10

If $f(x)$ is continuous on $[a, b]$ and $f(a) < y < f(b)$ then there exists at least one value, $x = c$, on (a, b) such that $f(c) = y$

IVT

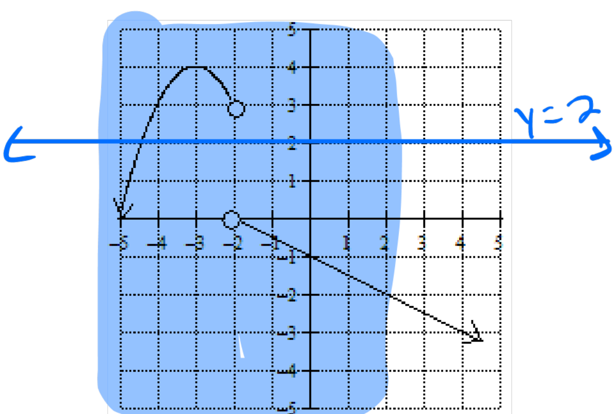
I. f(x) continuous on [a,b]

II f(a) < f(c) < f(b)



Now, investigate the graphs below to determine if the theorem is applicable for these functions on the specified intervals for the values given.

$$f(x) = \begin{cases} -(x+3)^2 + 4, & x < -2 \\ -\frac{1}{2}x - 1, & x > -2 \end{cases}$$



Is there a value of *c* on [−5, 2] such that *f*(*c*) = 2?

yes, c is between (-5,-4)

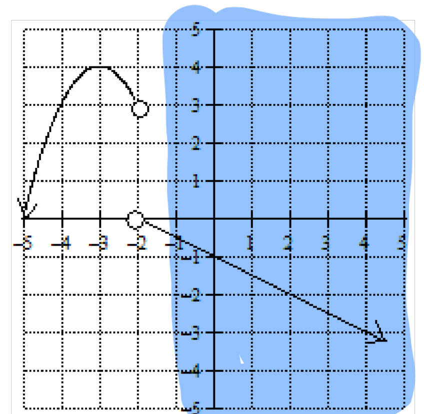
Does the I.V.T. guarantee a value of *c* such that *f*(*c*) = 2 on the interval [−5, 2]? Why or why not?

NO. The IVT does not guarantee a value of c such that f(c)=2 on [-5,2] b/c f(x) is not continuous on [-5,2]

I. f(x) is not continuous on [-5,2]

∴ The IVT does not guarantee a value of c such that f(c)=2 on [-5,2]

$$f(x) = \begin{cases} -(x+3)^2 + 4, & x < -2 \\ -\frac{1}{2}x - 1, & x > -2 \end{cases}$$



Is there a value of on *c* [−1, 5] such that *f*(*c*) = 2?

No, there is no c such that f(c)=2

Does the I.V.T. guarantee a value of *c* such that *f*(*c*) = 2 on the interval [−1, 5]? Why or why not?

I. f(x) is continuous on [-1,5]

II f(c)=2 is not between f(-1) and f(5)

∴ The IVT does not guarantee a value of c such that f(c)=2.

What two conditions must be true to verify the applicability of the Intermediate Value Theorem?

1. f(x) must be continuous on [a, b]
2. f(c) must be between f(a) and f(b)

For each of the following functions, determine if the I.V.T. is applicable or not and state why or why not. Then, if it is applicable, find the value of *c* guaranteed to exist by the theorem.

VA @ $x = -2$

1. $f(x) = \frac{x-3}{x+2}$ on the interval [–1, 3] for $f(c) = \frac{2}{3}$

I. $f(x)$ is continuous on $[-1, 3]$

II. $f(-1) = \frac{-1-3}{-1+2} = \frac{-4}{1} = -4$

$f(3) = \frac{3-3}{3+2} = \frac{0}{5} = 0$

$f(c) = 2/3$ is <u>not</u> between $f(-1)$ & $f(3)$

∴ The IVT does not apply for $f(c) = 2/3$ on $[-1, 3]$

VA @ $x = -2$

2. $f(x) = \frac{x-3}{x+2}$ on the interval [–4, 1] for $f(c) = \frac{2}{3}$

I. $f(x)$ is not continuous on $[-4, 1]$

∴ The IVT does not apply for $f(c) = 2/3$ on $[-4, 1]$

$Y_1 = e^{x+2}\cos(x)$

Table

3. $p(x) = e^{x+2}\cos x$ on the interval [−2, 1] for $p(c) = 5$

I. $p(x)$ is continuous on $[-2,1]$

II. $p(-2) = -1.131$

$p(1) = 10.852$

$p(1) < p(c) = 5 < p(1)$

∴ The IVT applies and guarantees a value of c on $(-2,1)$ such that $f(c)=5$

$f(x) = e^{x+2}\cos x \Leftarrow Y_1$

$5 = e^{c+2}\cos c \qquad Y_2 = 5$

$c \approx -0.334$

4. $f(x) = \frac{x}{x-2}$ on the interval [−1, 1] for $f(c) = -\frac{1}{2}$

I. $f(x)$ is continuous on $[-1,1]$

II. $f(-1) = \frac{-1}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$

$f(1) = \frac{1}{1-2} = \frac{1}{-1} = -1$

So $f(1) < f(c) = -\frac{1}{2} < f(-1)$

∴ The IVT guarantees a value of c on $(-1,1)$ such that $f(c) = -\frac{1}{2}$

$f(x) = \frac{x}{x-2}$

$\cancel{2}(c-2)\ -\frac{1}{\cancel{2}} = \frac{c}{\cancel{c-2}}\ \cancel{(c-2)}(2)$

$-c+2 = 2c$

$2 = 3c$

$\frac{2}{3} = c$

5. $f(x) = -\left(\frac{1}{2}\right)^{-x+3} - 2$ on the interval [3, 5] for $f(c) = -4$

I. $f(x)$ is continuous on $[3,5]$

II. $f(3) = -\left(\frac{1}{2}\right)^{-3+3} - 2 = -\left(\frac{1}{2}\right)^0 - 2 = -1-2 = -3$

$f(5) = -\left(\frac{1}{2}\right)^{-5+3} - 2 = -\left(\frac{1}{2}\right)^{-2} - 2 = -(2)^2 - 2 = -4-2 = -6$

So, $f(5) < f(c) = -4 < f(3)$

∴ The IVT applies and guarantees a value of c such that $f(c) = -4$

$f(x) = -\left(\frac{1}{2}\right)^{-x+3} - 2$

$-4 = -\left(\frac{1}{2}\right)^{-c+3} - 2$

$-2 = -\left(\frac{1}{2}\right)^{-c+3}$

$2 = \left(\frac{1}{2}\right)^{-c+3}$

$2^1 = (2)^{c-3}$

$1 = c-3$

$4 = c$