Differentiating $a^{f(x)}$ and $\log_a f(x)$

Notes 4.1 Name_____

Derivative of Exponential Functions with base a

$$\frac{d}{dx}a^u = a^u \cdot \ln a \cdot u'$$

- Assume u is a differentiable function.

Find the derivative of each function.

#1) $y = 3^{6x^2}$

$a=3$ $u=6x^2$
$u'=12x$

$y' = a^u \cdot \ln a \cdot u'$

$y' = 3^{6x^2} \cdot \ln 3 \cdot (12x)$

#2) $y = 8^{\tan x}$

$a=8$ $u=\tan x$
$u'=\sec^2 x$

$y' = a^u \cdot \ln a \cdot u'$

$y' = 8^{\tan x} \cdot \ln 8 \cdot (\sec^2 x)$

#3) If $y = e^{3x}$, what is the equation of the tangent line at $x = 0$?

Point $(0,1)$

$y = e^{3(0)}$

$y = e^0$

$y = 1$

Slope $m=3$

$y = e^{3x}$

$y' = a^u \cdot \ln a \cdot u'$

$a=e$ $u=3x$
$u'=3$

$y' = e^{3x} \cdot \ln e \cdot 3$

$y' = 3e^{3x}$

slope @ $x=0$

$y' = 3e^{3(0)}$

$y' = 3e^0$

$y' = 3 \cdot 1$

$y' = 3$

Point slope

$y - y_1 = m(x - x_1)$

$y - (0) = 3(x - (1))$

$y = 3x - 3$

Derivative of Logarithmic Functions with base a

$$\frac{d}{dx}\log_a u = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$$

- Assume u is a differentiable function.

Find the derivative of each function.

#4) $f(x) = \log_4(2x^5)$

$a=4$ $u=2x^5$
$u'=10x^4$

$f'(x) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$= \frac{1}{2x^5} \cdot \frac{1}{\ln 4} \cdot 10x^4$

$f'(x) = \frac{5}{x \ln 4}$

#5) $f(x) = \log_6(\cos x)$

$a=6$ $u=\cos x$
$u'=-\sin x$

$f'(x) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$f'(x) = \frac{1}{\cos x} \cdot \frac{1}{\ln 6}(-\sin x)$

$f'(x) = \frac{-\tan x}{\ln 6}$

#6) At what coordinate point(s) is the tangent line of $y = 1n(3x^2)$ parallel to $y = 3x - 4 \Rightarrow m=3$

① find f'

$f'(x) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$a=e$ $u=3x^2$
$u'=6x$

$f'(x) = \frac{1}{3x^2} \cdot \frac{1}{\ln e}(6x)$

$f'(x) = \frac{2}{x}$ ← slope of all tangent lines

② Set slope $= f'$

$3 = \frac{2}{x}$

$3x = 2$

$x = \frac{2}{3}$ ← x-value of point

③ Plug x into $f(x)$

$f(x) = \ln(3x^2)$

$f(\frac{2}{3}) = \ln[3(\frac{2}{3})^2]$

$f(\frac{2}{3}) = \ln[3 \cdot \frac{4}{9}]$

$f(\frac{2}{3}) = \ln(\frac{4}{3})$

$(\frac{2}{3}, \ln\frac{4}{3})$

Differentiating $a^{f(x)}$ and $\log_a f(x)$

Notes 4.1
Day 2 Combining Rules…

Name_____

#9) $y = \ln(\cos(x^2))$

$a = e$ $\quad u = \cos(x^2)$
$u' = -\sin(x^2)\,(x^2)'$
$u' = -2x\sin x^2$

$y' = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$= \frac{1}{\cos(x^2)} \cdot \frac{1}{\ln e} \cdot (-2x\sin x^2)$

$y' = -2x\tan x^2$

#10) $y = \log_6(3x\tan x)$

log Properties

$y = \log_6 3x + \log_6 \tan x$

$y' = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' + \frac{1}{u}\frac{1}{\ln a} u'$

$y' = \frac{1}{3x}\frac{1}{\ln 6} \cdot 3 + \frac{1}{\tan x} \cdot \frac{1}{\ln 6} \sec^2 x$

$y' = \frac{1}{x\ln 6}\frac{\tan x}{\tan x} + \frac{\sec^2 x}{\tan x \ln 6} \cdot \frac{x}{x}$

$y' = \frac{\tan x + x\sec^2 x}{x\ln 6 \tan x}$

#11) $f(x) = \frac{\log_3 x}{e^{3x}}$

$f'(x) = \frac{(\log_3 x)'\, e^{3x} - \log_3 x\,(e^{3x})'}{(e^{3x})^2}$

$f'(x) = \frac{\left[\frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'\right] e^{3x} - \log_3 x\left[a^u \ln a \cdot u'\right]}{e^{6x}}$

$a = 3 \quad u = x$
$u' = 1$

$a = e \quad u = 3x$
$u' = 3$

$f'(x) = \frac{\left[\frac{1}{x} \cdot \frac{1}{\ln 3}(1)\right] e^{3x} - \log_3 x\left[e^{3x} \underbrace{\ln e}_{=1} \cdot 3\right]}{e^{6x}}$

$= \frac{e^{3x}\left[\frac{1}{x\ln 3} - 3\log_3 x\right]}{e^{6x}}$

$f'(x) = \frac{\frac{1}{x\ln 3} - 3\log_3 x}{e^{3x}}$

#13) $y = x^7 7^x$

$y' = (x^7)'\,7^x + x^7(7^x)'$

$y' = 7x^6 \cdot 7^x + x^7\left[a^u \cdot \ln a \cdot u'\right]$

$a = 7 \quad u = x$
$u' = 1$

$y' = 7x^6 \cdot 7^x + x^7\left[7^x \cdot \ln 7 \cdot 1\right]$

$y' = 7x^6 \cdot 7^x + x^7 \cdot 7^x \cdot \ln 7$

#14) $f(x) = e^{x\sin x}$

$a = e \quad u = x\sin x$
$u' = x'\sin x + x \cdot (\sin x)'$
$u' = 1 \cdot \sin x + x \cdot \cos x$
$u' = \sin x + x\cos x$

$f'(x) = a^u \cdot \ln a \cdot u'$

$= e^{x\sin x} \cdot \ln e \cdot (\sin x + x\cos x)$

$f'(x) = (\sin x + x\cos x)e^{x\sin x}$