

# Inverse Derivatives

Notes 4.2

## Inverse Trig Derivatives

$$\frac{d}{dx} \sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$$\frac{d}{dx} \sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot u'$$

$$\frac{d}{dx} \tan^{-1}(u) = \frac{1}{u^2+1} \cdot u'$$

- Assume  $u$  is a differentiable function.

Find the derivative.

$$\#1) \frac{d}{dx} \sin^{-1}(3x) = \frac{1}{\sqrt{1-u^2}} u'$$

$u = 3x \quad u' = 3$   
 $u^2 = 9x^2$

$$= \frac{3}{\sqrt{1-9x^2}}$$

$$\#2) \frac{d}{dx} \tan^{-1}(2x^2) = \frac{1}{u^2+1} \cdot u'$$

$u = 2x^2 \quad u' = 4x$   
 $u^2 = 4x^2$

$$= \frac{4x}{4x^2+1}$$

Name \_\_\_\_\_

## Inverse Trig Derivatives

$$\frac{d}{dx} \cos^{-1}(u) = -\frac{1}{\sqrt{1-u^2}} \cdot u'$$

$$\frac{d}{dx} \csc^{-1}(u) = -\frac{1}{|u|\sqrt{u^2-1}} \cdot u'$$

$$\frac{d}{dx} \cot^{-1}(u) = -\frac{1}{u^2+1} \cdot u'$$

- Assume  $u$  is a differentiable function.

Find the derivative.

$$\#3) \frac{d}{dx} \cos^{-1}(-10x) = -\frac{1}{\sqrt{1-u^2}} u'$$

$u = -10x$   
 $u^2 = 100x^2$   
 $u' = -10$

$$= -\frac{-10}{\sqrt{1-100x^2}}$$

$$= \frac{10}{\sqrt{1-100x^2}}$$

$$\#4) \frac{d}{dx} \csc^{-1}(5x^3) = -\frac{1}{|u|\sqrt{u^2-1}} u'$$

$u = 5x^3$   
 $u^2 = 25x^6$   
 $u' = 15x^2$

$$= -\frac{15x^2}{|5x^3|\sqrt{25x^6-1}}$$

$$= -\frac{3}{|x|\sqrt{25x^6-1}}$$

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## Derivative of an Inverse Function

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

#5) If  $f(x) = \cos x - 6x$  and  $f^{-1}(-9\pi) = \frac{3\pi}{2}$ , find the derivative of  $f^{-1}(x)$  at  $x = -9\pi$ .

$$f'(x) = -\sin x - 6$$

$$f^{-1}(-9\pi) = \frac{3\pi}{2}$$

$$\begin{aligned} \frac{1}{f'[f^{-1}(-9\pi)]} &= \frac{1}{-\sin\left[\frac{3\pi}{2}\right] - 6} \\ &= \frac{1}{-(-1) - 6} \\ &= \frac{1}{1 - 6} \\ &= \frac{1}{-5} \end{aligned}$$

#6) If  $f(x) = x^2$ , find the derivative of  $f^{-1}(x)$  at  $x = 4$ .

$$f'(x) = 2x$$

$$x = y^2$$

$$\pm\sqrt{x} = y$$

$$f^{-1}(x) = \sqrt{x}$$

Always use principle root.

$$f^{-1}(4) = \sqrt{4}$$

$$f^{-1}(4) = 2$$

$$\begin{aligned} \left. \frac{d}{dx} f^{-1}(x) \right|_{x=4} &= \frac{1}{f'[f^{-1}(4)]} \\ &= \frac{1}{2[2]} \end{aligned}$$

$$\left. \frac{d}{dx} f^{-1}(x) \right|_{x=4} = \frac{1}{4}$$

#7) What is the anti-derivative of  $\frac{12x}{\sqrt{1-36x^4}}$ ?

$$f(x) = \sin^{-1}(6x^2) + C$$

$$\begin{aligned} u^2 &= 36x^4 \\ u &= 6x^2 \\ u' &= 12x \end{aligned}$$