

4.3 L'Hôpital's Rule

Name: _____

Write your questions
and thoughts here!

Notes

Recall: Special Trig Limits $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ **Recall:** When evaluating limits, first try direct substitution! $\lim_{x \rightarrow 3} \frac{2x-5}{x} = \frac{2(3)-5}{3} = \frac{6-5}{3} = \frac{1}{3}$

Example 1: $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x - 2} = \frac{(2)^2 - 7(2) + 10}{(2) - 2} = \frac{4 - 14 + 10}{0} = \frac{0}{0}$

$= \lim_{x \rightarrow 2} \frac{(x-2)(x-5)}{x-2} = \lim_{x \rightarrow 2} (x-5) = (2) - 5 = -3$

L'Hôpital's Rule:

Suppose $f(a) = 0$ and $g(a) = 0$ and $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$. L'Hopital's Rule allows you to apply the following:

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Evaluate each limit. Use L'Hôpital's when possible.

2. $\lim_{x \rightarrow 2} \frac{x-2}{3x^3 - 6x^2 + x - 2} = \frac{0}{0}$

$= \lim_{x \rightarrow 2} \frac{1}{9x^2 - 12x + 1}$

$= \frac{1}{9(2)^2 - 12(2) + 1}$

$= \frac{1}{9(4) - 24 + 1}$

$= \frac{1}{36 - 23}$

$= \frac{1}{13}$

3. $\lim_{x \rightarrow 0} \frac{\sin(6x)}{x} = \frac{0}{0}$

$= \lim_{x \rightarrow 0} 6 \cos(6x)$

$= 6 \cos[6(0)]$

$= 6 \cos(0)$

$= 6 \cdot 1$

$= 6$

4. $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{1-1}{0} = \frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{0 - (-\sin(x))}{2x}$

$= \lim_{x \rightarrow 0} \frac{\sin(x)}{2x} = \frac{0}{0}$

$= \lim_{x \rightarrow 0} \frac{\cos(x)}{2}$

$= \frac{\cos(0)}{2}$

$= \frac{1}{2}$

L'HÔPITAL'S IS NOT THE QUOTIENT RULE!!

5. $\frac{d}{dx} \frac{\sin(6x)}{x} = \frac{[\sin(6x)]' \cdot x - \sin(6x) \cdot x'}{x^2}$

$= \frac{6 \cos(6x) \cdot x - \sin(6x) \cdot 1}{x^2}$

$= \frac{6x \cos(6x) - \sin(6x)}{x^2}$

Now
summarize
what you
learned!
