## Notes 6.4 – Properties of Definite Integrals

Given the integral statements, write what you think each is equivalent to. Be prepared to explain your reasoning with the rest of the class.

1. $\int_a^a f(x)dx = 0 = F(a) - F(a)$

2. Given that $a < c < b$, $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

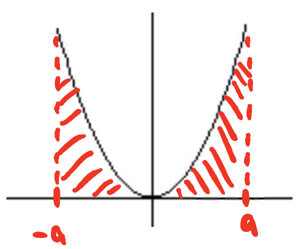
3. If $\int_a^b f(x)dx = K$, then $\int_b^a f(x)dx = -K$

4. Given that $b < a$, then $\int_a^b f(x)dx = -\int_b^a f(x)dx$

5. If $k$ is a constant, then $\int_a^b k \cdot f(x)dx = k\int_a^b f(x)dx$

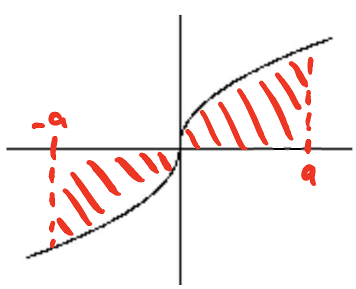
6. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

7. Given that $f(x)$ is an even function, $\int_{-a}^a f(x)dx = 2 \cdot \int_{-a}^0 f(x)dx = 2\int_0^a f(x)dx$




8. Given that $f(x)$ is an odd function, $\int_{-a}^a f(x)dx = \int_{-a}^0 f(x)dx + \int_0^a f(x)dx$

$= 0$

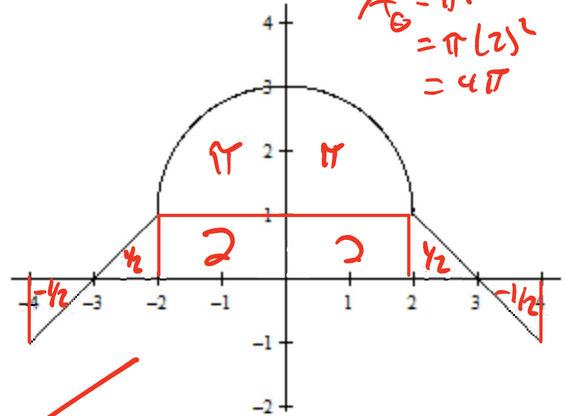


If $\int_0^3 f(x)dx = 6$ and $\int_3^7 f(x)dx = -8$, determine the value of each of the following integrals using the properties of definite integrals. Explain how you arrived at your answer for each.

| | |
|---|---|
| $\int_3^0 f(x)dx = -\int_0^3 f(x)dx$<br>$= -6$ | $\int_0^7 f(x)dx = \int_0^3 f(x)dx + \int_3^7 f(x)dx$<br>$= 6 + -8$<br>$= -2$ |
| $\int_3^3 f(x)dx = 0$ | $\int_7^3 3f(x)dx = -3\int_3^7 f(x)dx$<br>$= -3 \cdot (-8)$<br>$= 24$ |
| $\int_3^7 (2+3f(x))dx = \int_3^7 2dx + 3\int_3^7 f(x)dx$<br>$= 2x\Big\vert_3^7 + 3(-8)$<br>$= [2(7)] - [2(3)] - 24$<br>$= 14 - 6 - 24$<br>$= -16$ | $\int_{-3}^3 f(x)dx$, if $f(x)$ is an even function<br>$= 2\int_0^3 f(x)dx$<br>$= 2(6)$<br>$= 12$ |
| $\int_{-3}^3 f(x)dx$, if $f(x)$ is an odd function<br>$= \int_{-3}^0 f(x)dx + \int_0^3 f(x)dx$<br>$= -6 + 6$<br>$= 0$ | |

Pictured to the right is the graph of a function $f(x)$.

$A_{\circ} = \pi r^2$
$= \pi (2)^2$
$= 4\pi$




What is the value of $\int_0^3 f(x)\,dx$?

$= 2(1) + \frac{1}{4}\pi(2)^2 + \frac{1}{2}(1)(1)$

$= 2 + \pi + \frac{1}{2}$

$= 2.5 + \pi$

What is the value of $\int_0^4 f(x)\,dx$?

$= \int_0^3 f(x)dx + \int_3^4 f(x)dx$

$= (2.5+\pi) - (\frac{1}{2})$

$= 2 + \pi$

What is the value of $\int_{-3}^3 f(x)\,dx$?

$= 2 \cdot \int_0^3 f(x)dx$

$= 2(2.5+\pi)$

$= 5 + 2\pi$

If $F(0) = 5$, what is the value of $F(3)$, where $F$ is the anti-derivative of $f(x)$?

$\int_0^3 f(x)\,dx = F(x)\Big|_0^3$

$2.5+\pi = F(3) - F(0)$

$2.5+\pi = F(3) - 5$

$7.5+\pi = F(3)$

If $F(-2) = -2$, what is the value of $F(2)$, where $F$ is the anti-derivative of $f(x)$?

$\int_{-2}^2 f(x)\,dx = F(x)\Big|_{-2}^2$

$4 + 2\pi = F(2) - F(-2)$

$4 + 2\pi = F(2) - (-2)$

$4 + 2\pi = F(2) + 2$

$2 + 2\pi = F(2)$