Notes 6.5 – The Fundamental Theorem of Calculus, Particle Motion, and Average Value

Three Things to Always Keep In Mind:

1) $\int_a^b v(t)dt = p(b) - p(a)$
   where $v(t)$ represents the velocity and $p(t)$ represents the position.

2) Net Distance = $\int_a^b v(t)dt$
   The Net Distance the particle travels on the interval from $t = a$ to $t = \text{b}$. If $v(t) > 0$ on the interval (a, b), then it also represents the Total Distance.

3) Total Distance = $\int_a^b |v(t)|dt$
   The Total Distance the particle travels on the interval (a, b), whether or not $v(t) > 0$. To be safe, always do this integral when asked to find total distance when given velocity.

1. The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 0.5e^t(t-2)^3$. (This is a calculator active question.)

   a. If the position of the particle at $t = 1.5$ is 2.551, what is the position when $t = 3.5$?

$p(1.5) = 2.551$

FIND $P(3.5)$

$$\int_{1.5}^{3.5} 0.5e^t(t-2)^3\,dt = p(3.5) - p(1.5)$$

$$15.919 = p(3.5) - 2.551$$

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$$18.470 = p(3.5)$$

   b. What is the total distance that the object travels on the interval $t = 1$ to $t = 5$?

$$\text{Total Distance} = \int_1^5 \left|0.5e^t(t-2)^3\right|dt$$

$$= 913.067$$

2. The velocity of a particle that is moving along the x – axis is given by the function $v(t) = 3t^2 + 6$. (This is a non-calculator active question.)

   a. If the position of the particle at $t = 4$ is 72, what is the position when $t = 2$?

$p(4) = 72$

FIND $p(2)$

$$\int_2^4 v(t)dt = p(4) - p(2)$$

$$\int_2^4 (3t^2+6)dt = 72 - p(2)$$

$$(t^3+6t)\Big|_2^4 = 72 - p(2)$$

$$[4^3+6(4)] - [2^3+6(2)] = 72 - p(2)$$

$$64+24 - 8 - 12 = 72 - p(2)$$

$$68 = 72 - p(2)$$

$$-4 = -p(2)$$

$$p(2) = 4$$

   b. What is the total distance the particle travels on the interval $t = 0$ to $t = 7$?

$$\text{Total Distance} = \int_a^b |v(t)|dt$$

$$= \int_0^7 |3t^2+6|dt$$

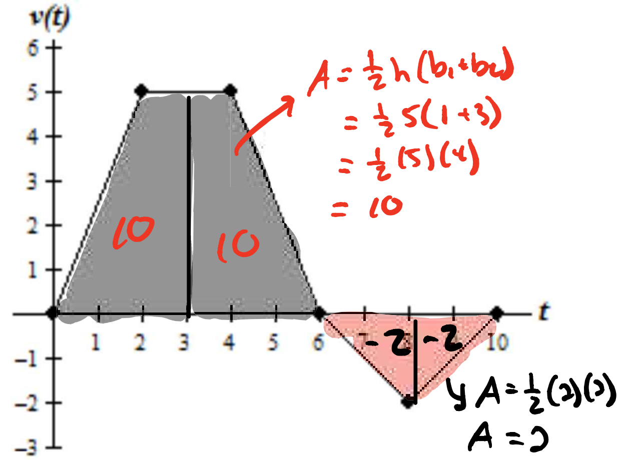
$$= |t^3+6t|\Big|_0^7$$

$$= |7^3+6(7)| - |0^3+6(0)|$$

$$= |343+42| - |0|$$

$$= 385$$

3. The graph of the velocity, measured in feet per second, of a particle moving along the x – axis is pictured. The position, $p(t)$, of the particle at $t = 8$ is 12. Use the graph of $v(t)$ to answer the questions that follow.



a. What is the position of the particle at $t = 3$?

$p(8) = 12$

$$\int_3^8 v(t)\,dt = p(8) - p(3)$$

$$\int_{3}^{6} v(t)\,dt + \int_6^8 v(t)\,dt = 12 - p(3)$$

$$10 + -2 = 12 - p(3)$$

$$8 = 12 - p(3)$$

$$-4 = -p(3)$$

$$4 = p(3)$$

b. What is the acceleration when $t = 5$?

$$a(5) = v'(5) = -\frac{5}{2}\ \text{ft/sec}^2$$

c. What is the net distance the particle travels from $t = 0$ to $t = 10$?

$$\text{Net Distance} = \int_0^{10} v(t)\,dt = \int_0^6 v(t)\,dt + \int_6^{10} v(t)\,dt$$

$$= 20 + -4$$

$$= 16 \text{ feet}$$

d. What is the total distance the particle travels from $t = 0$ to $t = 10$?

$$\text{Total Distance} = \int_0^{10} |v(t)|\,dt = \int_0^6 |v(t)|\,dt + \int_6^{10} |v(t)|\,dt$$

$$= |20| + |-4|$$

$$= 24 \text{ feet}$$

The table shows values of the velocity, $V(t)$ in meters per second, of a particle moving along the x – axis at selected values of time, t seconds.

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

a. What does the value of $\int_0^{18} V(t)dt$ represent?

The net distance the particle traveled from $t=0$ seconds to $t=18$ seconds.

b. Using a left Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

$\Delta x = 3$

$$\int_0^{18} v(t)dt = \Delta x\,(\text{Sum heights})$$
$$\approx 3[2.3 + 2.7 + 2.0 + 1.3 + 1.0 + 1.7] \approx 3[11] \approx 33 \text{ meters}$$

c. Using a right Riemann sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

$\Delta x = 3$

$$\int_0^{18} v(t)dt \approx 3[2.7 + 2.0 + 1.3 + 1.0 + 1.7 + 2.1]$$
$$\approx 3[10.8]$$
$$\approx 32.4 \text{ meters}$$

d. Using a midpoint Riemann sum of 3 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

$$\int_0^{18} v(t)dt \approx 6[2.7 + 1.3 + 1.7]$$

$\Delta x = 6$

$$\approx 6(5.7)$$
$$\approx 34.2 \text{ meters}$$

e. Using a trapezoidal sum of 6 subintervals of equal length, estimate the value of $\int_0^{18} V(t)dt$. Indicate units of measure.

$$\int_0^{18} v(t)dt \approx \tfrac{1}{2}(3)[2.3 + 2.7(2) + 2.0(2) + 1.3(2) + 1.0(2) + 1.7(2) + 2.1]$$
$$\approx \tfrac{3}{2}[21.8]$$
$$\approx 32.7 \text{ meters}$$

f. Find the average acceleration of the particle from $t = 3$ to $t = 9$. For what value of t, in the table, is this average acceleration approximately equal to $v'(t)$? Explain your reasoning.

$$AA = \frac{v(9)-v(3)}{9-3} = \frac{1.3-2.7}{6} = \frac{-1.4}{6} = -0.233 \text{ meters/second}^2$$

t	0	3	6	9	12	15	18
$V(t)$	2.3	2.7	2.0	1.3	1.0	1.7	2.1

The MVT guarantees a value of c on $(3,9)$ such that $v'(c) = \dfrac{f(3)-f(9)}{3-9}$

b/c
1. $v(t)$ is continuous on $[3,9]$
2. $v(t)$ is differentiable on $(3,9)$

At $t=6$, the average acceleration $\approx v'(t)$