Problems to Discuss before Quiz #4

Problem #1

Find the following limit. Explain the reasoning that you used to arrive at your answer.

$$\lim_{h \to 0} \frac{\cos 3(x+h) - \cos 3x}{h} = \boxed{-3\sin 3x}$$

The result of the limit should the derivative of the function $f(x) = \cos 3x$.

$$f'(x) = (-\sin 3x) \cdot 3$$

$$\boxed{f'(x) = -3\sin 3x}$$

Problem #2

Find the equation of the tangent line to the graph of the given function when $x = \frac{\pi}{3}$.

$$f(x) = 3x \cos x$$

$$f'(x) = 3 \cdot \cos x + 3x \cdot -\sin x$$

$$f'(x) = 3\cos x - 3x\sin x$$

$$f'(\pi/3) = 3\cos\frac{\pi}{3} - \cancel{3} \cdot \frac{\pi}{\cancel{3}} \cdot \sin\frac{\pi}{3}$$

$$= 3 \cdot \frac{1}{2} - \pi \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2} - \frac{\pi\sqrt{3}}{2}$$

$$= \frac{3 - \pi\sqrt{3}}{2} \quad \text{S.O.T}$$

$$f(\pi/3) = \cancel{3} \cdot \frac{\pi}{\cancel{3}} \cdot \cos\frac{\pi}{3} = \pi \cdot \frac{1}{2} = \frac{\pi}{2} \qquad \text{P.O.T}: \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\boxed{y - \frac{\pi}{2} = \frac{3 - \pi\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)}$$

Problem #3

Find the equation of the normal line to the graph of the function below when $x = -2$.

$$f(x) = \sqrt[3]{3x-2} = (3x-2)^{1/3}$$

$f'(x) = \frac{1}{3}(3x-2)^{-2/3} \cdot 3$

$f'(x) = \frac{1}{(3x-2)^{2/3}}$

$f'(-2) = \frac{1}{(3 \cdot -2 - 2)^{2/3}}$

$= \frac{1}{(-8)^{2/3}}$

$= \frac{1}{4}$

Slope of Normal: -4

$f(-2) = \sqrt[3]{3(-2)-2} = \sqrt[3]{-8} = -2$

P.O.T $= (-2, -2)$

$\boxed{y+2 = -4(x+2)}$

Problem #4

At what point on the graph of the function $f(x) = \sqrt{3x-2} = (3x-2)^{1/2}$ is the normal line perpendicular to the line defined by the equation $y = \frac{1}{4}x - 3$?

$f'(x) = \frac{1}{2}(3x-2)^{-\frac{1}{2}}(3)$

$= \frac{1}{2} \cdot \frac{1}{(3x-2)^{1/2}} \cdot \frac{3}{1}$

$= \frac{3}{2\sqrt{3x-2}} \Rightarrow$ Slope of the tangent

$\frac{3}{2\sqrt{3x-2}} = \frac{1}{4}$

$2\sqrt{3x-2} = 12$

$\sqrt{3x-2} = 6$

$3x - 2 = 36$

$3x = 38$

$x = \frac{38}{3}$

$f(\frac{38}{3}) = \sqrt{3 \cdot \frac{38}{3} - 2}$

$= \sqrt{36}$

$= 6$

$\boxed{\left(\frac{38}{3}, 6\right)}$
