4.1 Exp and Log Derivatives

Calculus Name: _____ **CA #1**

Find the derivative of each function.

1. $f(x) = e^{5x^2}$

$f'(x) = (5x^2)' \cdot e^{5x^2}$

$f'(x) = 10x\, e^{5x^2}$

3. $f(x) = e^{\sin(x^2)}$

$f'(x) = (\sin(x^2))' \cdot e^{\sin(x^2)}$

$= \cos(x^2)(x^2)' \cdot e^{\sin(x^2)}$

$= 2x\cos(x^2) \cdot e^{\sin(x^2)}$

5. $f(x) = \ln(x^3 - 2)$

$f'(x) = \dfrac{(x^3-2)'}{x^3-2}$

$= \dfrac{3x^2}{x^3-2}$

7. $f(x) = \log_2(x^3)$

$f'(x) = \dfrac{(x^3)'}{x^3} \cdot \dfrac{1}{\ln 2}$

$= \dfrac{3x^2}{x^3} \cdot \dfrac{1}{\ln 2}$

$f'(x) = \dfrac{3}{x \ln 2}$

9. $f(x) = \ln(\sec x + \csc x)$

$f'(x) = \dfrac{(\sec x + \csc x)'}{\sec x + \csc x}$

$f'(x) = \dfrac{\sec x \tan x - \csc x \cot x}{\sec x + \csc x}$

11. $f(x) = 3^{\sqrt{x}}$

$f'(x) = (\sqrt{x})' \cdot 3^{\sqrt{x}} \cdot \ln 3$

$= \frac{1}{2} x^{-\frac{1}{2}} \cdot 3^{\sqrt{x}} \cdot \ln 3$

$= \dfrac{\ln 3 \cdot 3^{\sqrt{x}}}{2\sqrt{x}}$

13. $f(x) = \ln(5^x)$

$f(x) = x \cdot \underbrace{\ln 5}_{\text{coefficient}}$ (Power Rule)

$f'(x) = \ln 5$

15. If $f(x) = e^{3x^4}$, what is the equation of the tangent line at $x = 0$.

Point (0,1)

$f(0) = e^{3(0)^4}$

$= 3^0$

$f(0) = 1$

Slope

$f'(x) = (3x^4)' \cdot e^{3x^4}$

$f'(x) = 12x^3 e^{3x^4}$

$f'(0) = 12(0)^3 e^{3(0)^4}$

$f'(0) = 0$

Point-Slope

$y - y_1 = m(x - x_1)$

$y - 1 = 0(x - 0)$

$y - 1 = 0$

$y = 1$

16. At what coordinate point(s) is the tangent line of $f(x) = e^{x^2}$ parallel to $y = -8$.

1) FIND f'

$f'(x) = (x^2)' \cdot e^{x^2}$

$f'(x) = 2x e^{x^2}$

$m = 0$

2) FIND x-value of point by set $f' = 0$

$0 = 2x e^{x^2}$

$0 = 2x$ or $0 = e^{x^2}$ DNE

$0 = x$

3) FIND y-value of point

$f(0) = e^{(0)^2}$

$= e^0$

$f(0) = 1$

$(0, 1)$

17. $f(x) = e^{\sin x}$ on the interval $0 < x < \pi$. On this interval, when will the average rate of change equal the instantaneous rate of change. [This is applying the Mean Value Theorem]

$ARC = \frac{f(\pi) - f(0)}{\pi - 0}$

$= \frac{e^{\sin\pi} - e^{\sin 0}}{\pi}$

$= \frac{e^0 - e^0}{\pi}$

$= \frac{0}{\pi}$

$ARC = 0$

$f'(x) = (\sin x)' e^{\sin x}$

$f'(x) = \cos x \, e^{\sin x}$

MVT

$ARC = f'(c)$

$0 = \cos c \cdot e^{\sin c}$

$0 = \cos c$ or $e^{\sin c} = 0$ DNE

$\frac{\pi}{2} = c$

18. Find the values of x where the tangent line to the graph of $y = \ln(x)$ is parallel to $5x + 3y = 9$

Slope of tangent

$y' = \frac{x'}{x}$

$y' = \frac{1}{x}$

$3y = -5x + 9$

$y = -\frac{5}{3}x + 9$

$m = -\frac{5}{3}$

$\parallel m = -\frac{5}{3}$

$\frac{1}{x} = -\frac{5}{3}$

$3 = -5x$

$\frac{3}{-5} = x$

Answers to 4.1 CA #1

1. $10xe^{5x^2}$		3. $2x\cos(x^2)\,e^{\sin(x^2)}$		5. $\frac{3x^2}{x^3-2}$	
7. $\frac{3}{x \ln 2}$		9. $\frac{\sec x \tan x - \csc x \cot x}{\sec x + \csc x}$		11. $\frac{\ln(3)3^{\sqrt{x}}}{2\sqrt{x}}$	
13. $\ln 5$	14. $\frac{5\cos(\ln(x^5))}{x}$	15. $y = 1$	16. $(0,1)$	17. $x = \frac{\pi}{2}$	18. $x = -\frac{3}{5}$