

4.2 Inverse Derivatives

Calculus

Name: _____

CA #1

Find the following.

$$1. \frac{d}{dx} \sin^{-1}(8x) = \frac{u'}{\sqrt{1-u^2}}$$

$$= \frac{8}{\sqrt{1-64x^2}}$$

$$3. \frac{d}{dx} \cot^{-1}(2x) = \frac{-u'}{u^2+1}$$

$$= \frac{-2}{4x^2+1}$$

$$5. \frac{d}{dx} \tan^{-1}(6x^3) = \frac{u'}{u^2+1}$$

$$= \frac{18x^2}{36x^6+1}$$

$$7. \frac{d}{dx} \csc^{-1}(x^3) = \frac{-u'}{|u|\sqrt{u^2-1}}$$

$$= \frac{-3x^2}{|x^3|\sqrt{x^6-1}}$$

$$= \frac{-3}{|x|\sqrt{x^6-1}}$$

9. Anti-derivative of $f'(x) = \frac{20x^4}{1+16x^{10}}$

$$f(x) = \tan^{-1}(4x^5) + C$$

11. Anti-derivative of $f'(x) = \frac{3}{\sqrt{1-9x^2}}$

$$f(x) = \sin^{-1}(3x) + C$$

$$13. \frac{d}{dx} \cos^{-1}(7x^3) = \frac{-u'}{\sqrt{1-u^2}}$$

$$= \frac{-21x^2}{\sqrt{1-49x^6}}$$

INVERSE FUNCTIONS:

15. If $f(x) = 4\sqrt{x}$ and $f^{-1}(20) = 25$, find the derivative of $f^{-1}(x)$ at $x = 20$

① $\frac{d}{dx} [f^{-1}(20)] = \frac{1}{f'(f^{-1}(20))}$ ② $f'(x) = 2x^{-\frac{1}{2}}$
 $= \frac{1}{f'(25)}$ $f'(25) = \frac{2}{\sqrt{25}}$
 $= \frac{1}{\frac{2}{5}}$ ③ $f'(25) = \frac{2}{5}$
 $= \frac{5}{2}$

16. If $f(x) = \cos(x)$ and $f^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$, find the derivative of $f^{-1}(x)$ at $x = \frac{\sqrt{2}}{2}$

$\frac{d}{dx} [f^{-1}\left(\frac{\sqrt{2}}{2}\right)] = \frac{1}{f'(f^{-1}\left(\frac{\sqrt{2}}{2}\right))}$ $f'(x) = -\sin x$
 $= \frac{1}{f'\left(\frac{\pi}{4}\right)}$ $f'\left(\frac{\pi}{4}\right) = -\sin\frac{\pi}{4}$
 $= \frac{1}{-\frac{\sqrt{2}}{2}}$ $f'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
 $= -\sqrt{2}$

17. If $f(x) = 3x + 10$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = -2$.

① $\frac{d}{dx} [f^{-1}(-2)] = \frac{1}{f'(f^{-1}(-2))}$ ② $x = 3y + 10$
 $= \frac{1}{f'(-4)}$ ③ $x - 10 = 3y$
 $= \frac{1}{3}$ ⑤ $\frac{x-10}{3} = y$
 $f^{-1}(x) = \frac{x-10}{3}$
 $f^{-1}(-2) = \frac{-2-10}{3}$
 $= \frac{-12}{3}$
 $f^{-1}(-2) = -4$

④ $f'(x) = 3$

18. If $f(x) = \frac{x^3}{27}$ find $\frac{d}{dx} [f^{-1}(x)]$ at $x = 1$.

① $\frac{d}{dx} [f^{-1}(1)] = \frac{1}{f'(f^{-1}(1))}$ ② $x = \frac{y^3}{27}$
 $= \frac{1}{f'(3)}$ ③ $27x = y^3$
 $= \frac{1}{3}$ ⑤ $3\sqrt[3]{x} = y$
 $f^{-1}(x) = 3\sqrt[3]{x}$
 $f^{-1}(1) = 3\sqrt[3]{1}$
 $= 3 \cdot 1$
 $= 3$

④ $f'(x) = \frac{1}{9}x^2$
 $f'(3) = \frac{1}{9}(3)^2$
 $= \frac{1}{9}(9)$
 $f'(3) = 1$

Answers to 4.2 CA #1

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|------------------------------------|--------------------------|------------------------|-----------------|----------------------------|-------|---------------------------------|
| 1. $\frac{8}{\sqrt{1-64x^2}}$ | | 3. $-\frac{2}{4x^2+1}$ | | 5. $\frac{18x^2}{36x^6+1}$ | | 7. $-\frac{3}{ x \sqrt{x^6-1}}$ |
| | 9. $\tan^{-1}(4x^5) + C$ | | | 11. $\sin^{-1}(3x) + C$ | | |
| 13. $\frac{21x^2}{\sqrt{1-49x^6}}$ | | 15. $\frac{5}{2}$ | 16. $-\sqrt{2}$ | 17. $\frac{1}{3}$ | 18. 1 | |