AP Calculus Test #2
Practice Multiple Choice

(All of these should be completed WITHOUT a calculator unless otherwise indicated.)

1. If $f(x) = 3x^3 - 2x^2 + 4$, what is the value of $\boxed{\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1}}$? $f'(-1)$

   A. 7   B. 10   C. 13   D. −1

   $f'(x) = 9x^2 - 4x$
   $f'(-1) = 9(-1)^2 - 4(-1)$
   $= 9(1) + 4$
   $= 13$

2. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximate value for $f(1.5)$ found by using the line tangent to the graph of $f(x)$ at $x = \frac{\pi}{2}$?

   A. −2   B. 1   C. $\pi - 2$ D. $4 - \pi$

   PoT $\left(\frac{\pi}{2}, 1\right)$

   $f\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{\pi}{2}\right) + 1$
   $= 2 \cdot 0 + 1$
   $= 0 + 1$
   $f\left(\frac{\pi}{2}\right) = 1$

   SoT: −2

   $f'(x) = -2\sin x$
   $f'\left(\frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}\right)$
   $= -2 \cdot (1)$
   $f'\left(\frac{\pi}{2}\right) = -2$

   Tangent line

   $y - 1 = -2\left(x - \frac{\pi}{2}\right)$
   $y = -2\left(1.5 - \frac{\pi}{2}\right) + 1$
   $y = -3 + \pi + 1$
   $y = -2 + \pi$

   $f(1.5) \approx \pi - 2$

3. Suppose the equation $x + 3 = -2(y - 1)$ is the equation of the normal line drawn to the graph of a function, $g(x)$, at $x = a$. Which of the following statements can be made about $g(x)$ at $x = a$?

   A. The graph of $g(x)$ is increasing at $x = a$. $-\frac{1}{2}(x+3) = y - 1$
   B. The graph of $g(x)$ is decreasing at $x = a$.   SoN $= -\frac{1}{2}$
   C. The graph of $g(x)$ has a horizontal tangent at $x = a$.   SoT $= 2$
   D. No conclusion can be made about the graph of $g(x)$ at $x = a$.

4. Let f be the function defined by $f(x) = x + 2\cos x$ and let g be defined by the function $g(x) = x^4 + x^3$. The line tangent to the graph of f at $x = \frac{3\pi}{2}$ is parallel to the line tangent to the graph of g at $x = a$. What is the value of a? (CALCULATOR)

A. −1　　B. 0.984　　C. 0.715　　D. 0.687

$$\boxed{\begin{aligned} f(x) &= x + 2\cos x \\ f' &= 1 - 2\sin x \end{aligned}}$$

$$\boxed{\begin{aligned} g(x) &= x^4 + x^3 \\ g' &= 4x^3 + 3x^2 \end{aligned}}$$

$$f'\left(\tfrac{3\pi}{2}\right) = g'(a)$$
$$1 - 2\sin\left(\tfrac{3\pi}{2}\right) = 4a^3 + 3a^2$$
$$1 - 2(-1) = 4a^3 + 3a^2$$
$$1 + 2 = 4a^3 + 3a^2$$
$$3 = 4a^3 + 3a^2$$

$Y_1 = 3$
$Y_2 = 4a^3 + 3a^2$

2nd CALC INTERSECT
$x = 0.715$

5. The graph of the derivative of a function, $f(x)$, is pictured to the right. Which of the following statements is/are not true?

I. The graph of f has relative extrema at $x = -2$, $x = 0$, and $x = 2$. T T False

II. The graph of f is decreasing on $(-\infty, -2) \cup (0, \infty)$. T

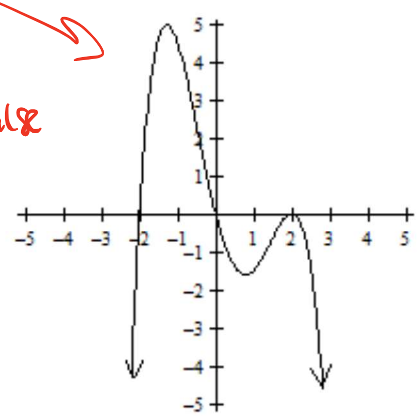
III. The slope of the normal line drawn to the graph of f at $x = -1$ has a negative value. T

A. III only　　B. I and II only

C. I and III only　　D. II only

Only I

Bad question



Graph of $f'(x)$

6. $\lim\limits_{h \to 0} \dfrac{\cos(x+h) - \cos x}{h} =$ $(\cos x)' = -\sin x$

A. $\sin x$　　B. $\cos x$　　C. 0　　D. $-\sin x$

7. If $f(x) = 2x^3 - \cos x$, then $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} =$ $f'(x)$

A. $6x^2 + \sin x$　　B. $6x + \sin x$　　C. $6x^2 - \sin x$　　D. does not exist

8. If $g(x) = \dfrac{2x}{\sqrt[3]{x}}$, what is the equation of the normal line drawn to $g(x)$ when $x = -8$?

A. $y - 8 = -\frac{2}{3}(x - 8)$

B. $y - 8 = \frac{3}{2}(x + 8)$

C. $y - 8 = \frac{3}{8}(x + 8)$

D. $y - 8 = -\frac{8}{3}(x + 8)$

$g(x) = 2x \cdot x^{-\frac{1}{3}} = 2x^{2/3}$

$g'(x) = \frac{4}{3}x^{-\frac{1}{3}}$

$g'(-8) = \dfrac{4}{3\sqrt[3]{-8}} = \dfrac{4}{3(-2)} = \dfrac{2}{-3}$

SOT

$\therefore$ SON: $3/2$

9. Which of the following functions are such that their graph is decreasing when $x = \pi$?

I. $f(x) = \dfrac{3}{\sqrt{x}} = 3 \cdot x^{-\frac{1}{2}}$

$f'(x) = -\frac{3}{2}x^{-3/2} = \dfrac{-3}{2\sqrt{x^3}}$

$f'(\pi) = \dfrac{-3}{2\sqrt{\pi^3}}$

$f'(\pi) =$ neg

II. $h(x) = \sqrt{x} + \sin x$

$h'(x) = \frac{1}{2}x^{-1/2} + \cos x = \frac{1}{2\sqrt{x}} + \cos x$

$h'(\pi) = \frac{1}{2\sqrt{\pi}} + \cos(\pi)$

$= \frac{1}{2\sqrt{\pi}} - 1$

$=$ (Less than 1) $- 1$

$h'(\pi) =$ neg

III. $g(x) = \dfrac{2}{x^2} + \cos x = 2x^{-2} + \cos x$

$g'(x) = -4x^{-3} - \sin x = \frac{-4}{x^3} - \sin x$

$g'(\pi) = \frac{-4}{\pi^3} - \sin \pi$

$= \frac{-4}{\pi} - 0$

$g'(\pi) =$ neg

A. I and II only B. II only C. I, II, and III D. II and III only

10. If $g(x) = \dfrac{x - 2x^2 + x^3}{x^4}$, then $g'(x) =$

A. $\dfrac{1 - 4x + 3x^2}{4x^3}$

B. $-\dfrac{3 - 4x + x^2}{x^4}$

C. $-\dfrac{3 - 4x}{x^2}$

D. $-\dfrac{3 + 4x + x^2}{x^4}$

$g(x) = x^{-3} - 2x^{-2} + x^{-1}$

$g'(x) = -3x^{-4} + 4x^{-3} - x^{-2}$

$= \frac{-3}{x^4} + \frac{4 \cdot x}{x^3 \cdot x} - \frac{1}{x^2} \cdot \frac{x^2}{x^2}$

FACTOR OUT -1

$= \dfrac{-3 + 4x - x^2}{x^4}$