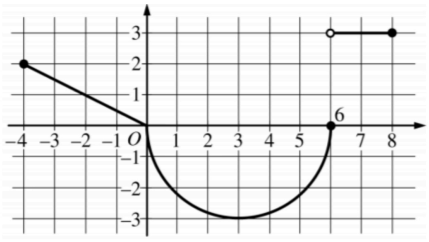
NO CALCULATOR IS ALLOWED FOR THIS QUESTION.



Graph of g

1. The function g is defined on the closed interval $[-4,8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)dt$

(a) Find $f(7)$ and $f'(x)$

Area under curve

$f(7) = 3(7) + \int_0^7 g(t)dt$

$f(7) = 21 - \frac{1}{2}\pi(3)^2 + 1(3)$ (Good Enough) +1

$f(7) = 24 - \frac{9}{2}\pi$

$f'(x) = 3 + g(x) \cdot x'$

$f'(x) = 3 + g(x)$

$f'(7) = 3 + g(7)$

$f'(7) = 3 + 3$ (Good Enough) +1

$f'(7) = 6$

(b) Find the value of x in the closed interval $[-4,3]$ at which f attains its maximum value. Justify your answer.

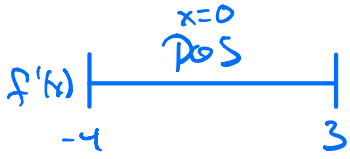
$f'(x) = 3 + g(x)$

$0 = 3 + g(x)$

$-3 = g(x)$

$\therefore$ $x=3$ is a critical value

$x=3$ is also an end value.



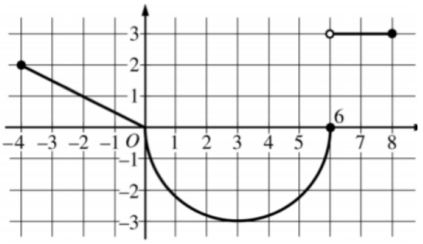
$\therefore f' > 0$ on $[-4,3]$

$\therefore f$ is increasing on $[-4,3]$ +1

$\therefore f(x)$ has a maximum at $x=3$ +1

$$f(x) = 3x + \int_0^x g(t)dt$$

(c) For each of $\lim_{x\to 0^-} g'(x)$ and $\lim_{x\to 0^+} g'(x)$, find the value or state that it does not exist.



$\lim_{x\to 0^-} g'(x) = -\frac{1}{2}$ +1

↑ what is the slope of $g(x)$ from left of 0?

$\lim_{x\to 0^+} g'(x)$ does not exist +1 (vertical tangent line)

↑ what is the 0^+ of $g'(x)$ from left of 0?

(d) Find $\lim_{x\to -2} \frac{f(x)+7}{e^{3x+6}-1}$

$\lim_{x\to -2}(f(x)+7) = \lim_{x\to -2}\left(3x + \int_0^x g(t)\,dt + 7\right)$

$= 3(-2) + \int_0^{-2} g(t)\,dt + 7$

$= -6 - \int_{-2}^{0} g(t)\,dt + 7$

$= -6 - 1 + 7$

$= 0$

$\lim_{x\to -2}\left(e^{3x+6} - 1\right) = e^{3(-2)+6} - 1$

$= e^0 - 1$

$= 1 - 1$

$= 0$

+1

L'Hopital's Rule

$\lim_{x\to -2} \frac{f(x)+7}{e^{3x+6}-1} = \lim_{x\to -2} \frac{f'(x)}{3e^{3x+6}} = \frac{f'(-2)}{3e^{3(-2)+6}} = \frac{3+g(-2)}{3e^0} = \frac{3+1}{3\cdot 1} = \frac{4}{3}$

+1 +1

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(x) = \begin{cases} \sqrt{9-x^2} & -3 \le x \le 0 \\ -x + 3\cos\left(\frac{\pi x}{2}\right) & 0 < x \le 4 \end{cases}$$

Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

$ARC = \dfrac{f(-3) - f(4)}{-3-4}$

$= \dfrac{0 - (-1)}{-7}$

$ARC = \dfrac{1}{-7}$

+1

SIDE WORK (NOT Required)

$f(-3) = \sqrt{9-(-3)^2}$
$= \sqrt{9-9}$
$= \sqrt{0}$
$f(-3) = 0$

$f(4) = -4 + 3\cos\left(\pi \cdot \frac{4}{2}\right)$
$= -4 + 3\cos(2\pi)$
$= -4 + 3 \cdot 1$
$f(4) = -1$

(b) Write an equation for the line tangent to the graph of f at $x = 3$

POT	SOT	Tangent line
$f(3) = -3 + 3\cos\left(\frac{3}{2}\pi\right)$ $f(3) = -3 + 3 \cdot 0$ $f(3) = -3$	$f'(x) = -1 - 3\sin\left(\frac{\pi}{2}x\right) \cdot \frac{\pi}{2}$ $f'(3) = -1 - \frac{3\pi}{2}\sin\left(\frac{3}{2}\pi\right)$ $= -1 - \frac{3\pi}{2}(-1)$ $f'(3) = -1 + \frac{3\pi}{2}$ +1	$y + 3 = \left(-1 + \frac{3\pi}{2}\right)(x-3)$ +1

(c) Find the average value of f on the interval $-3 \leq x \leq 4$ (CALC for this part)

$$\text{Average Value} = \frac{1}{4-(-3)}\int_{-3}^{4} f(x)\,dx$$

$$= \frac{1}{7}\left[\int_{-3}^{0} f(x)dx + \int_{0}^{4} f(x)dx\right] \quad +1$$

$$= \frac{1}{7}\left[\frac{9\pi}{4} - 8\right] \quad +1$$

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \leq x \leq 4$? Justify your answer.

I) $f(0) = 3$ $\therefore$ $f(x)$ is defined at $x=0$

II) $\lim_{x\to 0^-} f(x) = 3 = \lim_{x\to 0^+} f(x)$ $\therefore \lim_{x\to 0} f(x)$ exists

III $f(0) = \lim_{x\to 0} f(x)$

$\therefore$ $f(x)$ is continuous at $x=0$

$\therefore$ $f(x)$ is continuous on $[-4,3]$ $+1$

$\therefore$ The Extreme Value Theorem guarantees f attains an absolute maximum on $[-3,4]$ $+1$