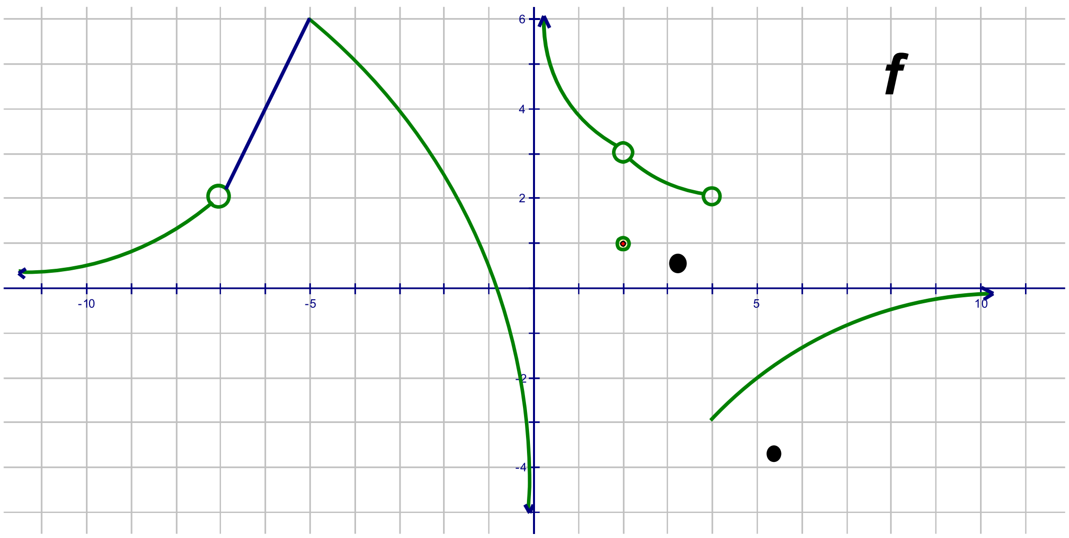
Unit 1 (Topics 1.1-1.16) Review – LIMITS and CONTINUITY

PART I - DO NOT USE A CALCULATOR ON ANY PROBLEM IN THIS SECTION. (Problems 1-37)

Consider the graph of function, f, shown below.



Answer the following questions about function f.

1.) $f(-5) =$ 6

2.) $f(2) =$ 1

3.) $f(4) =$ -3

4.) $\lim_{x\to -7} f(x) =$ 2

5.) $\lim_{x\to -5} f(x) =$ 6

6.) $\lim_{x\to 2} f(x) =$ 3

7.) $\lim_{x\to 4} f(x) =$ DNE

8.) $\lim_{x\to 0} f(x) =$ DNE

9.) $\lim_{x\to 0^-} f(x) =$ $-\infty$ (DNE)

10.) $\lim_{x\to 0^+} f(x) =$ ∞ (DNE

11.) $\lim_{x\to 4^+} f(x) =$ -3

12.) $\lim_{x\to 4^-} f(x) =$ 2

13.) $\lim_{x\to -\infty} f(x) =$ 0^+

14.) $\lim_{x\to \infty} f(x) =$ 0^-

15.) Use the definition of a continuous function at a number to answer the following.
Be sure to use reasons based on the definition of continuity at a point that we discussed in class.

a.) f is not continuous at x = -7 because: f(-7) is not defined

b.) f is not continuous at x = 2 because: $f(2) \neq \lim_{x\to 2} f(x)$

c.) f is not continuous at x = 4 because: $\lim_{x\to 4} f(x)$ dne

DO NOT USE A CALCULATOR

16.) $\lim\limits_{x\to 2}\left(-x^2+4x\right)$

$= -(2)^2 + 4(2)$

$= -4 + 8$

$= 4$

17.) $\lim\limits_{x\to 9^-}\dfrac{\sqrt{x}-3}{x-9}\cdot\dfrac{(\sqrt{x}+3)}{(\sqrt{x}+3)}$

$= \lim\limits_{x\to 9^-}\dfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$= \dfrac{1}{\sqrt{9}+3}$

$= \dfrac{1}{3+3}$

$= \dfrac{1}{6}$

18.) $\lim\limits_{x\to 0}\dfrac{x}{\tan x}$ produces indeterminate form $\frac{0}{0}$

$\lim\limits_{x\to 0} x = 0$, $\lim\limits_{x\to 0}\tan x = 0$ $\therefore$ L'HOSPITAL'S

$\lim\limits_{x\to 0}\dfrac{1}{\sec^2 x} = \dfrac{1}{(\sec 0)^2} = \dfrac{1}{1^2}$

$= 1$

$x=1$, $y=0$, $r=1$

$\cos 0 = 1$

19.) $\lim\limits_{x\to -2^+}\left(\dfrac{x}{x+2}\right) = -\infty$

VA @ $x=-2$

x	$\frac{x}{x+2}$
-1.9	$\frac{-1.9}{-1.9+2} = \frac{-}{+} = -$

20.) $\lim\limits_{x\to 0^-}\left(1+\dfrac{1}{x}\right) = 1-\infty = -\infty$

VA

x	$\frac{1}{x}$
$-.1$	$\frac{1}{-.1} = \frac{+}{-} = -$

21.) $\lim\limits_{x\to 1}\left(\sin \pi x\right) = \sin(\pi\cdot 1)$

$= \sin(\pi)$

$= 0$

$x=-1$, $y=0$, $r=1$

22.) $\lim\limits_{x\to\infty}\dfrac{7-6x^5}{x+3} = -\infty$ FASTER

OR

$= \lim\limits_{x\to\infty}\dfrac{-6x^5}{x}$

$= \lim\limits_{x\to\infty} -6x^4$

$= -\infty$

23.) $\lim\limits_{t\to -\infty}\dfrac{6-t^3}{7t^3+3} = \dfrac{-1}{7}$ same, same

OR $\lim\limits_{x\to -\infty}\dfrac{-t^3}{7t^3} = \lim\limits_{x\to -\infty}\dfrac{-1}{7} = \dfrac{-1}{7}$

24.) $\lim\limits_{x\to -\infty}\dfrac{x-2}{x^2+2x+1} = 0$ faster

or $= \lim\limits_{x\to -\infty}\dfrac{x}{x^2} = \lim\limits_{x\to -\infty}\dfrac{1}{x} = 0$

25.) $\lim\limits_{y\to -\infty}\dfrac{2-y}{\sqrt{7+6y^2}}$

$= \lim\limits_{y\to -\infty}\dfrac{\overset{+}{-y}}{\underset{+}{\sqrt{6}\,|y|}} = \dfrac{1}{\sqrt{6}}$

26.) $\lim\limits_{x\to 2} f(x)$ when

$$f(x)=\begin{cases} x^2-3x+6, & x < 2\\ -x^2+3x+2, & x \ge 2\end{cases}$$

$\lim\limits_{x\to 2^-}(x^2-3x+6) = 2^2-3(2)+6 = 4$

$\lim\limits_{x\to 2^+}(-x^2+3x+2) = -(2)^2+3(2)+2 = 4$

$\therefore \lim\limits_{x\to 2} f(x) = 4$

27.) If a $\neq$ 0, then $\lim\limits_{x\to -a}\dfrac{x^2-a^2}{x^4-a^4}$ is:

$= \lim\limits_{x\to -a}\dfrac{x^2-a^2}{(x^2-a^2)(x^2+a^2)}$

$= \lim\limits_{x\to a}\dfrac{1}{x^2+a^2} = \dfrac{1}{(a)^2+a^2}$

$= \dfrac{1}{2a^2}$

28.) Find a *c* such that $f(x)$ is continuous on the entire real line.

$$f(x)=\begin{cases} x^2 & \text{when} \quad x \le 4 \\ \dfrac{c}{x} & \text{when} \quad x > 4 \end{cases}$$

$\lim\limits_{x\to 4^-}(x^2) = \lim\limits_{x\to 4^+} \frac{c}{x}$

$4^2 = \frac{c}{4}$

$64 = c$

29.) Find the *x*-values (if any) at which *f* is discontinuous. Label as removable or non-removable.

$$f(x)=\frac{2x+6}{2x^2-18} = \frac{2\cancel{(x+3)}}{2\cancel{(x+3)}(x-3)}$$

f(x) has a removable discontinuity at x = -3 and a non-removable discontinuity at x = 3

30.) Determine all of the vertical asymptotes of *f*(*x*):

$$f(x)=\frac{x+2}{x^2-4} = \frac{\cancel{x+2}}{(x-2)\cancel{(x+2)}}$$

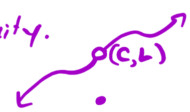
VA @ x = 2

31.) True or False: If f is undefined at *x* = *c*, then the limit of *f*(*x*) as *x* approaches c does not exist.

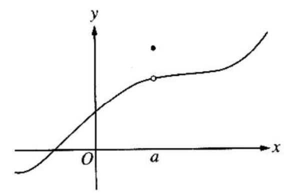
FALSE, the $\lim\limits_{x\to c} f(x)$ could exist if there is a hole at x = c.

32.) True or False: If the $\lim\limits_{x\to c} f(x) = L$ then $f(c) = L$.

False, the graph could have a removable discontinuity.



33.) The graph of the function *f* is shown to the right. Which of the following statements is false?



a.) $x = a$ is in the domain of f T

b.) $\lim\limits_{x\to a^+} f(x)$ is equal to $\lim\limits_{x\to a^-} f(x)$ T

c.) $\lim\limits_{x\to a} f(x)$ exists T

d.) $\lim\limits_{x\to a} f(x)$ is not equal to $f(a)$ T

e.) f is continuous at $x = a$ FALSE

34.) $\lim\limits_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{x^2} + 2x\cdot\Delta x + \Delta x^2 - \cancel{2x} - 2\Delta x + \cancel{1} - \cancel{x^2} + \cancel{2x} - \cancel{1}}{\Delta x}$

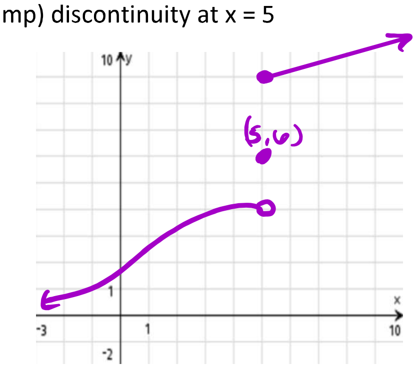
$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{\Delta x}(2x + \Delta x - 2)}{\cancel{\Delta x}}$ (GCF Δx)

$= \lim\limits_{\Delta x\to 0} (2x + \Delta x - 2) = 2x + 0 - 2$

$= 2x - 2$

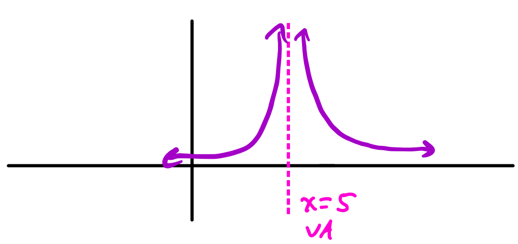
35.) On the graph, draw a function that has the following properties:

- A step (or jump) discontinuity at x = 5
- *f*(5) = 6.



36.) Create a function such that the $\lim\limits_{x\to 5}$ does not exist because it is approaching $+\infty$ from both the left and the right. Show both the function and the graph.

$f(x) = \dfrac{1}{(x-5)^2}$



37.) Find a function *f*(*x*) such that *f*(*x*) has a hole at *x* = 7 and a vertical asymptote at *x* = -4.

$f(x) = \dfrac{x-7}{(x+4)(x-7)}$

Unit 1 (Topics 1.1-1.16) Review – LIMITS and CONTINUITY

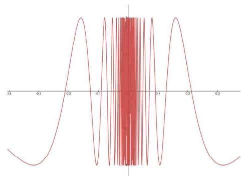
PART II - CALCULATORS MAY BE USED ON THE FIRST PART OF THIS SECTION.

1.) Approximate the limit ***numerically*** by completing the table:

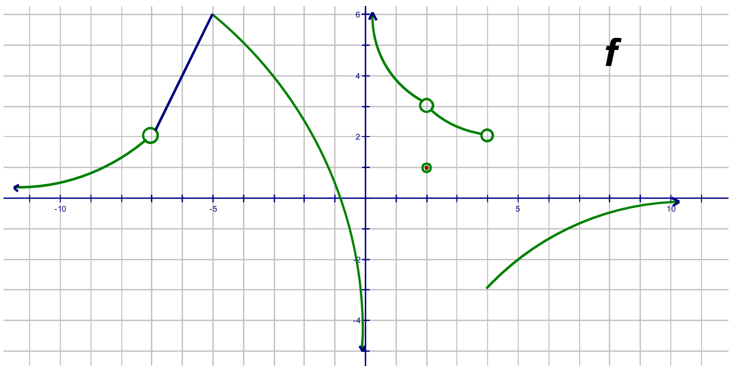
$$\lim_{x\to 2}\frac{x^2}{x-2} = \underline{\text{dne}}$$

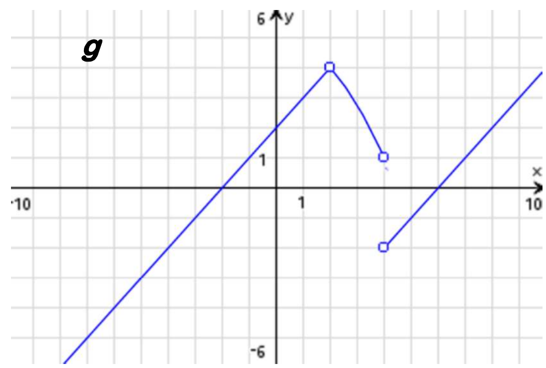
x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	−36.1	−396.01	−3996.001	und	4004.001	404.01	44.1

2.) Find the limit: $\lim_{x\to 0}\left(\cos\frac{1}{x}\right)$ DNE



oscillates





3.) Find $\lim_{x\to 2} f(g(x))$

$$= \lim_{x\to 4^-} f(x) = 2$$

4.) $\lim_{x\to 1} f(x-1)\cdot g(x)$

$$\lim_{x\to 1^-} f(x-1) \cdot \lim_{x\to 1^-} g(x) = -\infty \cdot 3 = -\infty$$

$$\lim_{x\to 1^+} f(x-1) \cdot \lim_{x\to 1^+} g(x) = \infty \cdot 3 = \infty$$

$$\therefore \lim_{x\to 1} f(x-1)g(x) \text{ dne}$$

5.) $\lim_{x\to 1^+}\frac{f(x+1)}{g(x+3)}$ $= \dfrac{\lim_{x\to 2^+} f(x)}{\lim_{x\to 4^+} g(x)} = \dfrac{3}{-2}$

6.) No calculator. The piecewise function for $g(x)$ is below. Find the values for *a*, *b*, *c*, and *d* that make $f(x)$ continuous everywhere. Be sure to use the definition of continuity and demonstrate proper notation.

$$f(x)=\begin{cases}\dfrac{x^2+x-2}{x-1}, & x<1\\ a, & x=1\\ b(x-c)^2, & 1<x<4\\ d, & x=4\\ 2x-8, & x>4\end{cases}$$

① $\lim\limits_{x\to1^-}\dfrac{(x+2)(x-1)}{x-1}=\lim\limits_{x\to1^-}(x+2)=1+2=3 \therefore a=3$

$\lim\limits_{x\to1^+} b(x-c)^2 = b(1-c)^2$

$\therefore b(1-c)^2=3$

② $\lim\limits_{x\to4^-} b(x-c)^2 = b(4-c)^2$

$\lim\limits_{x\to4^+}(2x-8)=2(4)-8=0 \quad \therefore d=0$

$\therefore b(4-c)^2=0$

③ System to find b, c

$b(4-c)^2=0$

$b=0$ or $c=4$

④ If $b=0$, then $b(1-c)^2=3$

$0(1-c)^2=3$

$0=3$ FALSE

So $b\neq0$

⑤ If $c=4$, then $b(1-c)^2=3$

$b(1-4)^2=3$

$b(-3)^2=3$

$9b=3$

$b=\frac{1}{3}$ True

⑥ $\boxed{\therefore\ a=3,\ b=\frac{1}{3},\ c=4,\ d=0}$