# Unit 3.1 The Chain Rule

## Topic Question 5

Chain

Let $f(x) = x^3$ and $g(x) = \frac{x}{x-1}$. If $h$ is the function defined by $h(x) = f(g(x))$, which of the following gives a correct expression for $h'(x)$?

(A) $3(g(x))^2 = 3\left(\frac{x}{x-1}\right)^2$

(B) $3(g'(x))^2 = 3\left(\frac{(x-1)-x}{(x-1)^2}\right)^2$

(C) $3(g(x))^2 g'(x) = 3\left(\frac{x}{x-1}\right)^2 \cdot \frac{(x-1)-x}{(x-1)^2}$

(D) $(g'(x))^3 = \left(\frac{(x-1)-x}{(x-1)^2}\right)^3$

$$h(x) = \left(\frac{x}{x-1}\right)^3$$

$$h'(x) = \underbrace{3\left(\frac{x}{x-1}\right)^2 \cdot \frac{1(x-1) - x(1)}{(x-1)^2}}_{\text{Chain}} \Big\} \text{ Quotient}$$

## Topic Question 6

Chain

Let $f$ be the function defined by $f(x) = e^{h(x)}$, where $h$ is a differentiable function. Which of the following is equivalent to the derivative of $f$ with respect to $x$?

(A) $e^{h(x)}$

(B) $e^{h'(x)}$

(C) $h'(x)e^{h(x)}$

(D) $h(x)e^{h(x)-1}$

$$f(x) = e^{h(x)}$$

$$f'(x) = \underbrace{e^{h(x)} \cdot h'(x)}_{\text{Chain}}$$

## Topic Question 7 Chain

If $f(x) = \underbrace{(\underbrace{e^{3x}}_{\text{Chain}} + \underbrace{\sin(2x)}_{\text{Chain}})^4}_{\text{Chain}}$, then $f'(x) =$

(A) $4(3e^{3x} + 2\cos(2x))^3$

(B) $4(e^{3x} + \sin(2x))^3 (e^{3x} + \cos(2x))$

(C) $4(e^{3x} + \sin(2x))^3 (3e^{3x} + 2\sin(2x))$

(D) $4(e^{3x} + \sin(2x))^3 (3e^{3x} + 2\cos(2x))$

$$f'(x) = \underbrace{4(e^{3x} + \sin(2x))^3 \cdot (\underbrace{e^{3x} \cdot 3}_{\text{Chain}} + \underbrace{\cos(2x) \cdot 2}_{\text{Chain}})}_{\text{Chain}}$$

# Unit 3.1 The Chain Rule

## Topic Question 8

Chain

Let $f$ be the function defined by $f(x) = \sin(h(x))$, where $h$ is a differentiable function. Which of the following is equivalent to the derivative of $f$ with respect to $x$?

Ⓐ $\cos(h(x))$

Ⓑ $\cos(h'(x))$

Ⓒ $\cos(h(x))h'(x)$

Ⓓ $\sin(h(x))h'(x)$

$$\underbrace{f'(x) = \cos(h(x)) \cdot h'(x)}_{\text{Chain}}$$

## Topic Question 9

Chain

If $f(x) = \left(\underbrace{\cos(\sqrt{x})}_{\text{Chain}} - \underbrace{\ln(x^2)}_{\text{Chain}}\right)^3$, then $f'(x) =$

$$\underbrace{3(\cos\sqrt{x} - \ln x^2)^2 \cdot \left(\underbrace{-\sin\sqrt{x} \cdot \tfrac{1}{2}x^{-\frac{1}{2}}}_{\text{Chain}} - \underbrace{\tfrac{1}{x^2} \cdot 2x}_{\text{Chain}}\right)}_{\text{Chain}}$$

$$= 3(\cos\sqrt{x} - \ln x^2)^2 \cdot \left(-\sin\sqrt{x} \cdot \tfrac{1}{2\sqrt{x}} - \tfrac{2}{x}\right)$$

Ⓐ $3\left(-\frac{1}{2\sqrt{x}}\sin(\sqrt{x}) - \frac{2}{x}\right)^2$

Ⓑ $3(\cos(\sqrt{x}) - \ln(x^2))^2 \cdot \left(-\sin(\sqrt{x}) - \frac{1}{x^2}\right)$

Ⓒ $3(\cos(\sqrt{x}) - \ln(x^2))^2 \cdot \left(\frac{1}{2\sqrt{x}}\cos(\sqrt{x}) - \frac{2}{x}\right)$

Ⓓ $3(\cos(\sqrt{x}) - \ln(x^2))^2 \cdot \left(-\frac{1}{2\sqrt{x}}\sin(\sqrt{x}) - \frac{2}{x}\right)$

## Topic Question 10

Chain Chain

Let $f(x) = 5x^4$ and $g(x) = e^{2x} + x$. If $h$ is the function defined by $h(x) = f(g(x))$, which of the following gives a correct expression for $h'(x)$?

Ⓐ $20(g(x))^3 = 20(e^{2x} + x)^3$

Ⓑ $20(g'(x))^3 = 20(2e^{2x} + 1)^3$

Ⓒ $20(g(x))^3 \cdot g'(x) = 20(e^{2x} + x)^3 \cdot (2e^{2x} + 1)$

Ⓓ $5(g'(x))^4 = 5(2e^{2x} + 1)^4$

$h(x) = 5(g(x))^4$

$$\underbrace{h'(x) = 20(g(x))^3 \cdot g'(x)}_{\text{Chain}}$$

or

$h(x) = 5(e^{2x} + x)^4$

$$\underbrace{h'(x) = 20(e^{2x} + x)^3 \cdot (\underbrace{e^{2x} \cdot 2}_{\text{Chain}} + 1)}_{\text{Chain}}$$

# Unit 3.1 The Chain Rule

## APClassroom MC 2

If $f(x) = \ln(e^{2x})$, then $f'(x) =$

Ⓐ 1

Ⓑ 2

Ⓒ 2x

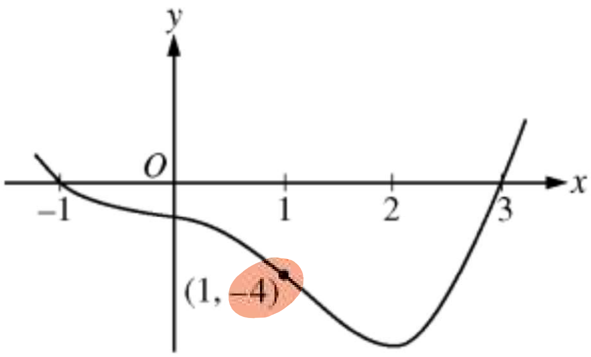
Ⓓ $e^{-2x}$

Ⓔ $2e^{-2x}$

Simplify first → $f(x) = \ln(e^{2x}) = 2x$

$f'(x) = 2$

## APClassroom FRQ 5




Graph of $f'$

Let $f$ be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of $f'$, the derivative of $f$, is shown above. The graph of $f'$ crosses the $x$-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let $g$ be the function given by $g(x) = e^{f(x)}$. chain

(3 points)

Write an equation for the line tangent to the graph of $g$ at $x = 1$.

| POT $(1, e^2)$ | SOT | Tangent |
|---|---|---|
| $g(1) = e^{f(1)}$ <br> $g(1) = e^2$ +1 | $g'(x) = \underbrace{f'(x) \cdot e^{f(x)}}_{\text{Chain}}$ <br> $g'(1) = f'(1) \cdot e^{f(1)}$ <br> $g'(1) = -4e^2$ +1 | $y - e^2 = -4e^2(x-1)$ +1 |

# Unit 3.1 The Chain Rule

## APCLASSROOM MC 24

If $y = \left(\frac{x}{x+1}\right)^5$, then $\frac{dy}{dx} =$ $\underbrace{5\left(\frac{x}{x+1}\right)^4 \cdot \underbrace{\frac{1\cdot(x+1)-x(1)}{(x+1)^2}}_{\text{Quotient}}}_{\text{Chain}}$

$= \frac{5x^4}{(x+1)^4}\cdot\frac{x+1-x}{(x+1)^2}$

$= \frac{5x^4}{(x+1)^6}$

(A) $5(1+x)^4$

(B) $\frac{x^4}{(x+1)^4}$

(C) $\frac{5x^4}{(x+1)^4}$

(D) $\frac{5x^4}{(x+1)^6}$

(E) $\frac{5x^4(2x+1)}{(x+1)^6}$

## APCLASSROOM MC 25

If $y = (x^3+1)^2$, then $\frac{dy}{dx} =$ $\underbrace{2(x^3+1)\cdot(3x^2)}_{\text{Chain}}$

$= 6x^2(x^3+1)$

(A) $(3x^2)^2$

(B) $2(x^3+1)$

(C) $2(3x^2+1)$

(D) $3x^2(x^3+1)$

(E) $6x^2(x^3+1)$

## APCLASSROOM MC 27

If $f(x) = e^{\tan^2 x}$, then $f'(x) =$ $\underbrace{e^{\tan^2 x}\cdot\underbrace{2\tan x\cdot\sec^2 x}_{\text{Chain}}}_{\text{Chain}}$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x\sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

## APCLASSROOM MC 28

If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$ $\frac{d}{dx}\left[(\ln x)^2 + 2(\ln x)\right]$

$= \underbrace{2\ln x\cdot\frac{1}{x}}_{\text{Chain}} + 2\cdot\frac{1}{x}$

$= \frac{2\ln x}{x} + \frac{2}{x}$

$= \frac{2\ln x + 2}{x}$

(A) $\frac{2\ln x+2}{x}$

(B) $2x\ln x + 2x$

(C) $2\ln x + 2$

(D) $2\ln x + \frac{2}{x}$

(E) $\frac{2x+2}{x}$

## APCLASSROOM MC 29

If $f(x) = e^{(2/x)}$, then $f'(x) =$ $\overbrace{e^{2x^{-1}}\cdot(-2x^{-2})}^{\text{Chain}}$

$= -\frac{2}{x^2}e^{2/x}$

(A) $2e^{(2/x)}\ln x$

(B) $e^{(2/x)}$

(C) $e^{(-2/x^2)}$

(D) $-\frac{2}{x^2}e^{(2/x)}$

(E) $-2x^2e^{(2/x)}$

## APCLASSROOM MC 32

If $y = \sin^3 x$, then $\frac{dy}{dx} =$ $\underbrace{3\sin^2 x\cdot\cos x}_{\text{Chain}}$

(A) $\cos^3 x$

(B) $3\cos^2 x$

(C) $3\sin^2 x$

(D) $-3\sin^2 x\cos x$

(E) $3\sin^2 x\cos x$

# Unit 3.1 The Chain Rule

## APCLASSROOM MC 35

If $f(x) = \cos^3(4x)$, then $f'(x) =$ $\underbrace{3\cos^2(4x)\underbrace{(-\sin(4x))\cdot 4}_{\text{Chain}}}_{\text{Chain}}$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$ $\quad = -12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

## APCLASSROOM MC 36

If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$

(A) $\frac{3\sqrt{3}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $-\frac{\sqrt{3}}{2}$

(D) $-\frac{3}{2}$

(E) $-\frac{3\sqrt{3}}{2}$

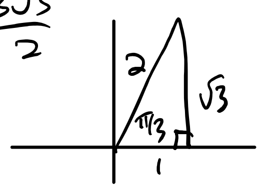
$f'(x) = \underbrace{-\sin(3x)\cdot 3}_{\text{Chain}}$

$f'\left(\frac{\pi}{9}\right) = -\sin\left(3\cdot\frac{\pi}{9}\right)\cdot 3$

$= -\sin\left(\frac{\pi}{3}\right)\cdot 3$

$= -\frac{\sqrt{3}}{2}\cdot 3$

$= \frac{-3\sqrt{3}}{2}$




## APCLASSROOM MC 48

If f is a differentiable function and $y = \sin(f(x^2))$ what is $\frac{dy}{dx}$ when $x = 3$?

(A) $\cos(f'(9))$

(B) $6\cos(f(9))$

(C) $f'(9)\cos(f(9))$

(D) $6f'(9)\cos(f(9))$

$y' = \underbrace{\cos(f(x^2))\cdot \underbrace{f'(x^2)\cdot 2x}_{\text{Chain}}}_{\text{Chain}}$

$y'(3) = \cos(f(3^2))\cdot f'(3^2)\cdot 2(3)$

$= \cos f(9)\cdot f'(9)\cdot 6$

## APCLASSROOM MC 52

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| –2 | –6 | 9 | –10 | 16 |
| 1 | 5 | –3 | 3 | –2 |
| 3 | 0 | 7 | 8 | 3 |

If $h(x) = f(g(x))$, what is the value of $h'(1)$?

(A) -19

(B) -14

(C) 7

(D) 9

$h' = f'(g(x))\cdot g'(x)$ Chain

$h'(1) = f'(g(1))\cdot g'(1)$

$= f'(3)\cdot(-2)$

$= 7(-2)$

$h'(1) = -14$

## APCLASSROOM MC 53CALCULATOR

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| –1 | 6 | 5 | 3 | –2 |
| 1 | 3 | –3 | –1 | 2 |
| 3 | 1 | –2 | 2 | 3 |

If $h(x) = f(g(x))$, what is the value of $h'(1)$?

(A) 5

(B) 6

(C) 9

(D) 10

(E) 12

$h' = f'(g(x))\cdot g'(x)$ Chain

$h'(1) = f'(g(1))\cdot g'(1)$

$= f'(-1)\cdot(2)$

$= 5(2)$

$h'(1) = 10$

## APCLASSROOM MC 116

The slope of the line tangent (derivative) to the graph of $y = \ln(1-x)$ at x = -1 is

(A) -1

(B) -1/2

(C) 1/2

(D) ln 2

(E) 1

$y' = \underbrace{\frac{1}{1-x}\cdot(-1)}_{\text{Chain}}$

$y'(-1) = \frac{1}{1-(-1)}(-1)$

$= \frac{1}{2}(-1)$

$y'(-1) = -\frac{1}{2}$

## APCLASSROOM FRQ 120

| $x$ | $-2$ | $-2 < x < -1$ | $-1$ | $-1 < x < 1$ | $1$ | $1 < x < 3$ | $3$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 12 | Positive | 8 | Positive | 2 | Positive | 7 |
| $f'(x)$ | $-5$ | Negative | 0 | Negative | 0 | Positive | $\frac{1}{2}$ |
| $g(x)$ | $-1$ | Negative | 0 | Positive | 3 | Positive | 1 |
| $g'(x)$ | 2 | Positive | $\frac{3}{2}$ | Positive | 0 | Negative | $-2$ |

(2 points)

The twice-differentiable functions *f* and *g* are defined for all real numbers *x*. The function *h* is defined by *h*(*x*)=ln(*f*(*x*)). Find *h*'(3). Show the computations that lead to your answer.

$$h'(x) = \overbrace{\frac{1}{f(x)} \cdot f'(x)}^{\text{Chain}} \quad +1$$

$$h'(3) = \frac{1}{f(3)} \cdot f'(3) = \frac{1}{7} \cdot \frac{1}{2} = \frac{1}{14} \quad +1$$

## APCLASSROOM FRQ 129

(5 points)

Let *f* be the function defined by $f(x) = (1 + \tan x)^{3/2}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$.
Write an equation for the line tangent to the graph of *f* at the point where $x = 0$.

| PoT | SoT | Tangent |
|---|---|---|
| $f(0) = (1 + \tan 0)^{3/2}$ | $f'(x) = \underbrace{\frac{3}{2}(1+\tan x)^{1/2} \sec^2 x}_{\text{chain}}$ +2 | $y - 1 = \frac{3}{2}(x - 0)$ +1 |
| $= (1+0)^{3/2}$ | | |
| $f(0) = 1$ +1 | $f'(0) = \frac{3}{2}\sqrt{1+\tan 0}\ \sec^2(0)$ | |
| | $= \frac{3}{2}(1)(1)^2$ | |
| | $f'(0) = \frac{3}{2}$ +1 | |