Unit 3.2 Implicit Differentiation

TOPIC QUESTION 3

Consider the curve defined by $xy^2 - 2x^3 = 2$ for $y \geq 0$.

(a) Show that $\frac{dy}{dx} = \frac{6x^2 - y^2}{2xy}$.

$$\underbrace{\frac{d}{dx}[xy^2]}_{\text{Product}} - \frac{d}{dx}(2x^3) = \frac{d}{dx}2$$

$$1 \cdot y^2 + x \cdot 2y\frac{dy}{dx} - 6x^2 = 0 \quad +1$$

$$2xy\frac{dy}{dx} = 6x^2 - y^2$$

$$\frac{dy}{dx} = \frac{6x^2 - y^2}{2xy} \quad +1$$

(b) Write an equation for the line tangent to the curve at the point $(1, 2)$. POT

SOT

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{6(1)^2 - (2)^2}{2(1)(2)}$$

$$= \frac{2}{4}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{2} \quad +1$$

Tangent line

$$y - 2 = \frac{1}{2}(x - 1) \quad +1$$

(c) Find the x-coordinate of the point P at which the line tangent to the curve at P is horizontal. $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 0$

$$0 = \frac{6x^2 - y^2}{2xy}$$

Can't solve → $6x^2 - y^2 = 0$ +1

$6x^2 = y^2$

need system of equations

$6x^2 = y^2$	$xy^2 - 2x^3 = 2$

$$x(6x^2) - 2x^3 = 2$$

$$6x^3 - 2x^3 = 2$$

$$4x^3 = 2$$

$$x^3 = \frac{1}{2}$$

$$x = \sqrt[3]{\frac{1}{2}} \quad +1$$

(d) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.

$$\frac{d^2y}{dx^2} = \frac{\left(12x - 2y\frac{dy}{dx}\right)(2xy) - (6x^2 - y^2)\overbrace{\left(2 \cdot y + 2x\frac{dy}{dx}\right)}^{\text{Product}}}{(2xy)^2}$$

Quotient +2

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = \frac{\left(12(1) - 2(2)\frac{1}{2}\right)(2(1)(2)) - (6(1)^2 - (2)^2)\left(2(2) + 2(1) \cdot \frac{1}{2}\right)}{(2(1)(2))^2}$$

$$= \frac{10 \cdot 4 - (2)(5)}{16}$$

$$= \frac{30}{16}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(1,2)} = 15/8$$

+1

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Topic Question 5

If $y = \ln(3x+4y)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{3x+4y}$

(B) $\frac{3}{3x+4y}$

(C) $\frac{7}{3x+4y}$

(D) $\frac{3}{3x+4y-4}$

$$\frac{dy}{dx} = \frac{1}{3x+4y}\left(3+4\frac{dy}{dx}\right)$$

$$(3x+4y)\frac{dy}{dx} = 3+4\frac{dy}{dx}$$

$$(3x+4y)\frac{dy}{dx} - 4\frac{dy}{dx} = 3$$

$$\frac{dy}{dx}(3x+4y-4) = 3$$

$$\frac{dy}{dx} = \frac{3}{3x+4y-4}$$

Topic Question 6

If $e^y - e^{y^2} = x - x^3$, then the value of $\frac{dy}{dx}$ at the point $(0,1)$ is

(A) $-\frac{1}{e}$

(B) $\frac{e-1}{2e}$

(C) $\frac{1+2e}{e}$

(D) undefined

$$e^y\frac{dy}{dx} - e^{y^2}2y\frac{dy}{dx} = 1-3x^2$$

$$\frac{dy}{dx}\left(e^y - 2ye^{y^2}\right) = 1-3x^2$$

$$\frac{dy}{dx} = \frac{1-3x^2}{e^y - 2ye^{y^2}}$$

$$\left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1-3(0)^2}{e^1 - 2(1)e^{1^2}} = \frac{1}{e-2e} = \frac{1}{-e}$$

Topic Question 7

If $\underbrace{2xy^2}_{\text{product}} - \underbrace{3x^2y}_{\text{Product}} = \underbrace{6x}_{\text{Power}}$, then $\frac{dy}{dx} =$

(A) $\frac{6}{4y-6x}$

(B) $\frac{2y^2-2xy-6}{3x^2}$

(C) $\frac{-2y^2+6xy+6}{4xy-3x^2}$

(D) $\frac{-2y^2+6xy+3x^2+6}{4xy}$

$$2\cdot y^2 + 2x\cdot 2y\frac{dy}{dx} - \left[6x\cdot y + 3x^2\frac{dy}{dx}\right] = 6$$

$$4xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 6 - 2y^2 + 6xy$$

$$\frac{dy}{dx}\left(4xy - 3x^2\right) = 6 - 2y^2 + 6xy$$

$$\frac{dy}{dx} = \frac{6-2y^2+6xy}{4xy-3x^2}$$

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Topic Question 8

If $3x^2 + 5x^2y^2 = 2y$, then $\frac{dy}{dx} =$

Power Product Power

(A) $\frac{-6x}{20xy-2}$

(B) $\frac{-10xy^2-6x}{10x^2y-2}$

(C) $\frac{2-6x-10xy^2}{10x^2y}$

(D) $\frac{6x+10xy^2+10x^2y}{2}$

$6x + 10x \cdot y^2 + 5x^2 \cdot 2y\frac{dy}{dx} = 2 \cdot \frac{dy}{dx}$

$6x + 10xy^2 = 2\frac{dy}{dx} - 10x^2y\frac{dy}{dx}$

$6x + 10xy^2 = \frac{dy}{dx}(2 - 10x^2y)$

$\frac{6x + 10xy^2}{2 - 10x^2y} = \frac{dy}{dx}$

$\frac{-6x - 10xy^2}{-2 + 10x^2y} = \frac{dy}{dx}$

Topic Question 9

If $y = \ln(2x^2 - 3y^2)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{2x^2-3y^2}$

(B) $\frac{4x}{2x^2-3y^2}$

(C) $\frac{4x-6y}{2x^2-3y^2}$

(D) $\frac{4x}{2x^2-3y^2+6y}$

$\frac{dy}{dx} = \frac{1}{2x^2-3y^2}\left(4x - 6y\frac{dy}{dx}\right)$

$(2x^2-3y^2)\frac{dy}{dx} = 4x - 6y\frac{dy}{dx}$

$(2x^2-3y^2)\frac{dy}{dx} + 6y\frac{dy}{dx} = 4x$

$\frac{dy}{dx}(2x^2 - 3y^2 + 6y) = 4x$

$\frac{dy}{dx} = \frac{4x}{2x^2-3y^2+6y}$

Topic Question 10

If $e^{2y} - e^{(y^2-y)} = x^4 - x^2$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 2

$e^{2y} \cdot 2 \cdot \frac{dy}{dx} - e^{y^2-y}\left(2y\frac{dy}{dx} - 1\frac{dy}{dx}\right) = 4x^3 - 2x$

$2e^{2y}\frac{dy}{dx} - e^{y^2-y}(2y-1)\frac{dy}{dx} = 4x^3 - 2x$

@ $(1,0)$ $2e^{2(0)}\frac{dy}{dx} - e^{0^2-0}(2(0)-1)\frac{dy}{dx} = 4(1)^3 - 2(1)$

$2 \cdot 1\frac{dy}{dx} - 1(-1)\frac{dy}{dx} = 2$

$2\frac{dy}{dx} + 1\frac{dy}{dx} = 2$

$3\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{3}$

7. Find (a) the equation of the tangent line and (b) the equation of the normal line drawn to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$ at the point (8, 1).

SoT: $\frac{1}{2}$ SoN: -2

$\frac{d}{dx} x^{2/3} + \frac{d}{dx} y^{2/3} = \frac{d}{dx} 5$

$\frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}} \frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{y}} \frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}}$

$\frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}} \cdot \frac{3\sqrt[3]{y}}{2}$

$\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$

$\left.\frac{dy}{dx}\right|_{(8,1)} = -\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$

Tangent Line

$y - 1 = -\frac{1}{2}(x - 8)$

Normal Line

$y - 1 = +2(x - 8)$