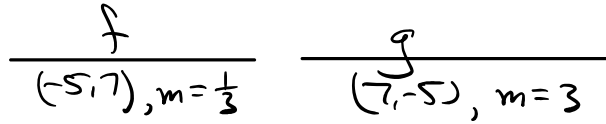


### Unit 3.3 Differentiating Inverse Functions

#### TOPIC QUESTION 5

Let  $f$  and  $g$  be inverse functions that are differentiable for all  $x$ . If  $f(-5) = 7$  and  $g'(7) = 3$ , which of the following statements must be false?

- I.  $f'(3) = -\frac{1}{3}$  **False**
- II.  $f'(-5) = \frac{1}{3}$  **True**
- III.  $f'(7) = \frac{1}{3}$  **Could be true**



- (A) I only
- (B) II only
- (C) III only
- (D) I and III only

In order for  $f$  to have an inverse,  
 $f$  must be always increasing b/c  $g'(7) > 0$   
 $\therefore f' > 0$   
 $f'(3) < 0$

#### TOPIC QUESTION 6

$\therefore$  inverse exists

For which of the following increasing functions  $f$  does  $(f^{-1})'(20) = \frac{1}{5}$ ?

(A)  $f(x) = x + 5$   
 $f'(x) = 1$   
 $f^{-1}(x) = 1$

$(f^{-1})'(20) = \frac{1}{5} \Rightarrow f(0) = 20$   
 $f'(0) = 5$

(B)  $f(x) = x^3 + 5x + 20$   
 $f' = 3x^2 + 5$   
 $f(0) = 20$   
 $5 = 3x^2 + 5$

(C)  $f(x) = x^5 + 5x + 14$

$0 = 3x^2$   
 $0 = x^2$   
 $0 = x$

$f' = 5x^4 + 5$   
 $5 = 5x^4 + 5$   
 $0 = 5x^4$   
 $0 = x$

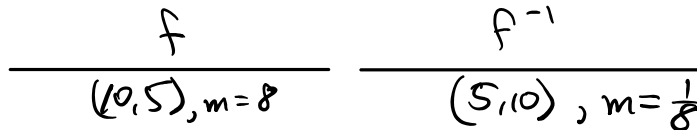
(D)  $f(x) = e^x + 5x + 19$   
 $f' = e^x + 5$   
 $5 = e^x + 5$   
 $0 = e^x$  no solution

#### TOPIC QUESTION 7

An increasing function  $f$  satisfies  $f(10) = 5$  and  $f'(10) = 8$ . Which of the following statements about the inverse of  $f$  must be true?

$\hookrightarrow$  INVERSE EXISTS.

(A)  $(f^{-1})'(5) = 10$



(B)  $(f^{-1})'(8) = 10$

(C)  $(f^{-1})'(5) = 8$

(D)  $(f^{-1})'(5) = \frac{1}{8}$

### Unit 3.3 Differentiating Inverse Functions

#### TOPIC QUESTION 8

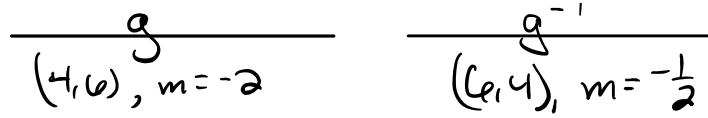
A decreasing function  $g$  satisfies  $g(4) = 6$  and  $g'(4) = -2$ . Which of the following statements about the inverse of  $g$  must be true?  
 ↳ INVERSE EXISTS.

(A)  $(g^{-1})'(6) = 4$

(B)  $(g^{-1})'(-2) = 4$

(C)  $(g^{-1})'(6) = -2$

(D)  $(g^{-1})'(6) = -\frac{1}{2}$



#### TOPIC QUESTION 9

For which of the following decreasing functions  $f$  does  $(f^{-1})'(10) = -\frac{1}{8}$ ?  
 ↳ INVERSE EXISTS.

(A)  $f(x) = -5x + 15$   $f'(x) = -5$

$(f^{-1})'(10) = -\frac{1}{8}$   $f'( ) = -5$   
 $f(10) =$   $f( ) = 10$

(B)  $f(x) = -2x^3 - 2x + 14$   $f' = -6x^2 - 2$   $f(1) = 10$

(C)  $f(x) = -x^5 - 4x + 15$

$f' = -5x^4 - 4$   
 $-8 = -5x^4 - 4 \rightarrow x^4 = \frac{4}{5}$   
 $x = \sqrt[4]{\frac{4}{5}}$

$f(\sqrt[4]{\frac{4}{5}}) \neq 10$

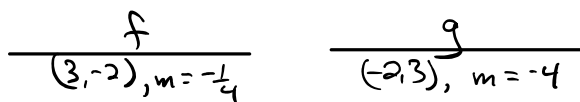
$f'(x) = -2e^{-2x} - 1$   $f(\frac{1}{2} \ln \frac{7}{2}) \neq 10$   
 $-8 = -2e^{-2x} - 1$   
 $-7 = -2e^{-2x}$   
 $\frac{7}{2} = e^{-2x}$   
 $\ln \frac{7}{2} = -2x$   
 $-\frac{1}{2} \ln \frac{7}{2} = x$

(D)  $f(x) = e^{-2x} - x + 9$

#### TOPIC QUESTION 10

Let  $f$  and  $g$  be inverse functions that are differentiable for all  $x$ . If  $f(3) = -2$  and  $g'(-2) = -4$ , which of the following statements must be false?

- I.  $f'(0) = \frac{1}{4}$  False
- II.  $f'(3) = -\frac{1}{4}$  True
- III.  $f'(5) = -\frac{1}{4}$  Maybe true



(A) I only

(B) II only

(C) III only

(D) I and III only

$g'(-2) = -4$  and  $g$  is the inverse of  $f$   
 $\therefore g$  is always decreasing.  
 $\therefore f$  is always decreasing.

### Unit 3.3 Differentiating Inverse Functions

#### APCLASSROOM MC 3 *Continuous*

Let  $f$  be a differentiable function such that  $f(3) = 15$ ,  $f(6) = 3$ ,  $f'(3) = -8$ , and  $f'(6) = -2$ . The function  $g$  is differentiable and  $g(x) = f^{-1}(x)$  for all  $x$ . What is the value of  $g'(3)$ ?

(A)  $-\frac{1}{2}$

(B)  $-\frac{1}{8}$

(C)  $\frac{1}{6}$

(D)  $\frac{1}{3}$

(E) The value of  $g'(3)$  cannot be determined from the information given.

$f$	$g$
$(3, 15), m = -8$	$(15, 3), m = -\frac{1}{8}$
$(6, 3), m = -2$	$(3, 6), m = -\frac{1}{2}$

#### APCLASSROOM MC 4 CALCULATOR PERMITTED

$x$	$f(x)$	$g(x)$	$f'(x)$
-4	0	-9	5
-2	4	-7	4
0	6	-4	2
2	7	-3	1
4	10	-2	3

The table above gives values of the differentiable functions  $f$  and  $g$ , and  $f'$ , the derivative of  $f$ , at selected values of  $x$ . If  $g(x) = f^{-1}(x)$ , what is the value of  $g'(4)$ ?

(A)  $-\frac{1}{3}$

(B)  $-\frac{1}{4}$

(C)  $-\frac{3}{100}$

(D)  $\frac{1}{4}$

(E)  $\frac{1}{3}$

$f$	$g$
$(-2, 4), m = 4$	$(4, -2), m = \frac{1}{4}$

### Unit 3.3 Differentiating Inverse Functions

#### APCLASSROOM MC 6

Let  $f$  be the function defined by  $f(x) = x^3 + x$ . If  $g(x) = f^{-1}(x)$  and  $g(2) = 1$ , what is the value of  $g'(2)$ ?

(A)  $\frac{1}{13}$

(B)  $\frac{1}{4}$

(C)  $\frac{7}{4}$

(D) 4

(E) 13

$$\frac{f}{(1,2), m=4} \quad \frac{g}{(2,1), m=\frac{1}{4}}$$

$$\begin{aligned} f'(x) &= 3x^2 + 1 \\ f'(1) &= 3(1)^2 + 1 \\ &= 3 + 1 \\ f'(1) &= 4 \end{aligned}$$

#### APCLASSROOM MC 7

Let  $f$  and  $g$  be functions that are differentiable everywhere. If  $g$  is the inverse function of  $f$  and if  $g(-2) = 5$  and  $f(5) = -1/2$ , then  $g'(-2) =$

(A) 2

(B)  $1/2$

(C)  $1/5$

(D)  $-\frac{1}{5}$

(E) -2

$$\frac{f}{(5, -\frac{1}{2})} \quad \frac{g}{(-2, 5), m=-2}$$

*Continuous*

#### APCLASSROOM MC 8 CALCULATOR PERMITTED

The functions  $f$  and  $g$  are differentiable. For all  $x$ ,  $f(g(x)) = x$  and  $g(f(x)) = x$ . If  $f(3) = 8$  and  $f'(3) = 9$ , what are the values of  $g(8)$  and  $g'(8)$ ?

(A)  $g(8) = \frac{1}{3}$  and  $g'(8) = -\frac{1}{9}$

(B)  $g(8) = \frac{1}{3}$  and  $g'(8) = \frac{1}{9}$

(C)  $g(8) = 3$  and  $g'(8) = -9$

(D)  $g(8) = 3$  and  $g'(8) = -\frac{1}{9}$

(E)  $g(8) = 3$  and  $g'(8) = \frac{1}{9}$

$$\frac{f}{(3,8), m=9} \quad \frac{g}{(8,3), m=\frac{1}{9}}$$

*INVERSES*

*Continuous*

### Unit 3.3 Differentiating Inverse Functions

APCLASSROOM MC 9 CALCULATOR PERMITTED

$x$	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

The table above gives selected values for a differentiable and increasing function  $f$  and its derivative. If  $g$  is the inverse function of  $f$ , what is the value of  $g'(3)$ ?

(A)  $\frac{1}{13}$

$$\frac{f}{(1,3), m=4} \quad \frac{g}{(3,1), m=\frac{1}{4}}$$

(B)  $\frac{1}{4}$

(C) 1

(D) 4

(E) 13

APCLASSROOM MC 14

Let  $f(x) = (2x + 1)^3$  and let  $g$  be the inverse function of  $f$ . Given that  $f(0) = 1$ , what is the value of  $g'(1)$ ?

(A)  $-\frac{2}{27}$

(B)  $\frac{1}{54}$

(C)  $\frac{1}{27}$

(D)  $\frac{1}{6}$

(E) 6

$$\frac{f}{(0,1), m=6} \quad \frac{g}{(1,0), m=\frac{1}{6}}$$

$$f' = 3(2x+1)^2 \cdot 2$$

$$f'(0) = 3(2 \cdot 0 + 1) \cdot 2$$

$$f'(0) = 6$$

### Unit 3.3 Differentiating Inverse Functions

APCLASSROOM FRQ 22 CALCULATOR PERMITTED

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions  $f$  and  $g$  are differentiable for all real numbers, and  $g$  is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of  $x$ . The function  $h$  is given by  $h(x) = f(g(x)) - 6$ .

If  $g^{-1}$  is the inverse function of  $g$ , write an equation for the line tangent to the graph of  $y = g^{-1}(x)$  at  $x = 2$ .

$g$   $g^{-1}$   
 $(1, 2), m = 5$   $(2, 1), m = \frac{1}{5}$   
 +1 pot sot +1  
 tangent line  
 $y - 1 = \frac{1}{5}(x - 2) + 1$

APCLASSROOM MC 24

Let  $f$  be the function defined by  $f(x) = 2x + e^x$ . If  $g(x) = f^{-1}(x)$  for all  $x$  and the point  $(0, 1)$  is on the graph of  $f$ , what is the value of  $g'(1)$ ?

(A)  $\frac{1}{2+e}$

(B)  $\frac{1}{3}$

(C)  $\frac{1}{2}$

(D) 3

(E)  $2 + e$

$f$   $g$   
 $(0, 1), m = 3$   $(1, 0), m = \frac{1}{3}$

$f'(x) = 2 + e^x$   
 $f'(0) = 2 + e^0$   
 $f'(0) = 3$

