Unit 3.3 Differentiating Inverse Functions

Topic Question 5

Let f and g be inverse functions that are differentiable for all x. If $f(-5) = 7$ and $g'(7) = 3$, which of the following statements must be false?

I. $f'(3) = -\frac{1}{3}$ False
II. $f'(-5) = \frac{1}{3}$ True
III. $f'(7) = \frac{1}{3}$ Could be true

f: $(-5,7), m=\frac{1}{3}$ g: $(7,-5), m=3$

(A) I only

(B) II only

(C) III only

(D) I and III only

In order for f to have an inverse, f must be always increasing b/c $g'(7)>0$
$\therefore f'>0$
$f'(3)<0$

Topic Question 6

$\therefore$ inverse exists

For which of the following increasing functions f does $(f^{-1})'(20) = \frac{1}{5}$?

$[f^{-1}]'(20) = \frac{1}{5} \Rightarrow f(0)=20$, $f'(0)=5$

(A) $f(x) = x + 5$ $f'(x)=1$, $f^{-1}(x)=1$

(B) $f(x) = x^3 + 5x + 20$ $f'=3x^2+5$ $f(0)=20$
$5=3x^2+5$
$0=3x^2$
$0=x^2$
$0=x$

(C) $f(x) = x^5 + 5x + 14$ $f'=5x^4+5$ $f(0)=14$
$5=5x^4+5$
$0=5x^4$
$0=x$

(D) $f(x) = e^x + 5x + 19$ $f'=e^x+5$
$5=e^x+5$
$0=e^x$ no solution

Topic Question 7

An increasing function f satisfies $f(10) = 5$ and $f'(10) = 8$. Which of the following statements about the inverse of f must be true?
↳ INVERSE EXISTS.

f: $(10,5), m=8$ f^{-1}: $(5,10), m=\frac{1}{8}$

(A) $(f^{-1})'(5) = 10$

(B) $(f^{-1})'(8) = 10$

(C) $(f^{-1})'(5) = 8$

(D) $(f^{-1})'(5) = \frac{1}{8}$

Unit 3.3 Differentiating Inverse Functions

Topic Question 8

A decreasing function g satisfies $g(4) = 6$ and $g'(4) = -2$. Which of the following statements about the inverse of g must be true?

↳ INVERSE EXISTS.

(A) $(g^{-1})'(6) = 4$

(B) $(g^{-1})'(-2) = 4$

(C) $(g^{-1})'(6) = -2$

(D) $(g^{-1})'(6) = -\frac{1}{2}$

g: $(4,6)$, $m=-2$

g^{-1}: $(6,4)$, $m=-\frac{1}{2}$

Topic Question 9

For which of the following decreasing functions f does $(f^{-1})'(10) = -\frac{1}{8}$?

↳ INVERSE EXISTS.

$[f^{-1}]'(10) = -\frac{1}{8}$

$f(10) =$

$f'(\ ) = -8$

$f(\ ) = 10$

(A) $f(x) = -5x + 15$ $\quad f'(x) = -5$

(B) $f(x) = -2x^3 - 2x + 14$

$f' = -6x^2 - 2$
$-8 = -6x^2 - 2$
$-6 = -6x^2$
$1 = x^2$
$1 = x$

$f(1) = 10$

(C) $f(x) = -x^5 - 4x + 15$

$f' = -5x^4 - 4$
$-8 = -5x^4 - 4$
$x^4 = \frac{4}{5}$
$x = \sqrt[4]{\frac{4}{5}}$

$f\left(\sqrt[4]{\frac{4}{5}}\right) \neq 10$

(D) $f(x) = e^{-2x} - x + 9$

$f'(x) = -2e^{-2x} - 1$
$-8 = -2e^{-2x} - 1$
$-7 = -2e^{-2x}$
$\frac{7}{2} = e^{-2x}$
$\ln\frac{7}{2} = -2x$
$-\frac{1}{2}\ln\frac{7}{2} = x$

$f(-\frac{1}{2}\ln\frac{7}{2}) \neq 10$

Topic Question 10

Let f and g be inverse functions that are differentiable for all x. If $f(3) = -2$ and $g'(-2) = -4$, which of the following statements must be false?

I. $f'(0) = \frac{1}{4}$ False
II. $f'(3) = -\frac{1}{4}$ True
III. $f'(5) = -\frac{1}{4}$ Maybe True

f: $(3,-2)$, $m=-\frac{1}{4}$

g: $(-2,3)$, $m=-4$

(A) I only

(B) II only

(C) III only

(D) I and III only

$g'(-2) = -4$ and g is the inverse of f

$\therefore$ g is always decreasing.

$\therefore$ f is always decreasing.

Unit 3.3 Differentiating Inverse Functions

APClassroom MC 3

Continuous

Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

Continuous

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

f
$(3, 15)$, $m = -8$
$(6, 3)$, $m = -2$

g
$(15, 3)$, $m = -1/8$
$(3, 6)$, $m = -\frac{1}{2}$

APClassroom MC 4 Calculator Permitted

x	$f(x)$	$g(x)$	$f'(x)$
−4	0	−9	5
−2	4	−7	4
0	6	−4	2
2	7	−3	1
4	10	−2	3

Continuous

The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$

(B) $-\frac{1}{4}$

(C) $-\frac{3}{100}$

(D) $\frac{1}{4}$

(E) $\frac{1}{3}$

f
$(-2, 4)$, $m = 4$

g
$(4, -2)$, $m = \frac{1}{4}$

Unit 3.3 Differentiating Inverse Functions

APCLASSROOM MC 6

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

f: $(1,2),\ m=4$ g: $(2,1),\ m=\frac{1}{4}$

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) $\frac{7}{4}$

(D) 4

(E) 13

$f'(x) = 3x^2+1$
$f'(1) = 3(1)^2+1$
$= 3+1$
$f'(1) = 4$

APCLASSROOM MC 7

Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -1/2$, then $g'(-2) =$

Continuous

f: $(5,-2) = -\frac{1}{2}$ g: $(-2,5),\ m=-2$

(A) 2

(B) 1/2

(C) 1/5

(D) $-\frac{1}{5}$

(E) -2

APCLASSROOM MC 8 CALCULATOR PERMITTED

INVERSES

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

Continuous

f: $(3,8),\ m=9$ g: $(8,3),\ m=\frac{1}{9}$

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

Unit 3.3 Differentiating Inverse Functions

APClassroom MC 9 Calculator Permitted

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

f: $(1,3)$, $m = 4$ g: $(3,1)$, $m = \frac{1}{4}$

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) 1

(D) 4

(E) 13

APClassroom MC 14

Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

f: $(0,1)$, $m = 6$ g: $(1,0)$, $m = \frac{1}{6}$

(A) $-\frac{2}{27}$

(B) $\frac{1}{54}$

(C) $\frac{1}{27}$

(D) $\frac{1}{6}$

(E) 6

$f' = 3(2x+1)^2 \cdot 2$

$f'(0) = 3(2 \cdot 0 + 1) \cdot 2$

$f'(0) = 6$

Unit 3.3 Differentiating Inverse Functions

APClassroom FRQ 22 Calculator Permitted

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

g: $(1,2), m=5$

g^{-1}: $(2,1), m=\frac{1}{5}$

+1 POT SOT +1

tangent line

$y-1=\frac{1}{5}(x-2)$ +1

APClassroom MC 24

Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point (0,1) is on the graph of f, what is the value of $g'(1)$?

- (A) $\frac{1}{2+e}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{2}$
- (D) 3
- (E) $2+e$

f: $(0,1), m=3$

g: $(1,0), m=\frac{1}{3}$

$f'(x) = 2+e^x$

$f'(0) = 2+e^0$

$f'(0) = 3$