Unit 3.4 Differentiating Inverse Trig Functions

Topic Question 1

$\frac{d}{dx}\left(\tan^{-1}(3x)\right) =$ $\frac{1}{1+(3x)^2} \cdot 3$

(A) $3\sec^2(3x)$

(B) $-3\csc^2(3x)$

(C) $\frac{3}{\sqrt{1-(3x)^2}}$

(D) $\frac{3}{1+(3x)^2}$

Topic Question 2

If $f(x) = \arctan x$, then $\lim_{x\to\sqrt{3}} \frac{f(x)-f(\sqrt{3})}{x-\sqrt{3}}$ is

$f'(\sqrt{3})$

$f'(x) = \frac{1}{1+x^2}$

$f'(\sqrt{3}) = \frac{1}{1+(\sqrt{3})^2}$

$f'(\sqrt{3}) = \frac{1}{4}$

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{\pi}{3}$

(D) nonexistent

Topic Question 3

$\frac{d}{dx}\left(\cot^{-1}x\right) =$ $-\frac{1}{1+x^2}$

(A) $-\csc^2 x$

(B) $\sec^2 x$

(C) $-\frac{1}{1+x^2}$

(D) $\frac{1}{1+x^2}$

Topic Question 6

If $f(x) = \arcsin x$, then $\lim_{x\to\frac{1}{2}} \frac{f(x)-f\left(\frac{1}{2}\right)}{x-\frac{1}{2}}$ is

$f'(x) = \frac{1}{\sqrt{1-x^2}}$

$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$

$= \frac{1}{\sqrt{1-\frac{1}{4}}}$

$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{\frac{3}{4}}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{2}{\sqrt{3}}$

(D) nonexistent

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APCLASSROOM MC 1

If $f(x) = \arccos(x^2)$, then f'(x)= $-\frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$

(A) $\frac{1}{\sqrt{1-x^4}}$

(B) $\frac{-2x}{\sqrt{1-x^4}}$

(C) $\frac{2x}{\sqrt{1-x^4}}$

(D) $\frac{-4x^3}{\sqrt{1-x^4}}$

(E) $\frac{4x^3}{\sqrt{1-x^4}}$

APCLASSROOM MC 2

If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx}$ = $\frac{1}{\sqrt{1-(5x)^2}} \cdot 5$

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

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APCLASSROOM MC 3

If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

IMPLICIT

(A) $\frac{y}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{1-x^2}} = \frac{1}{y}\frac{dy}{dx}$ chain

(B) $\frac{xy}{\sqrt{1-x^2}}$

$\frac{y}{\sqrt{1-x^2}} = \frac{dy}{dx}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

APCLASSROOM MC 4

If $\lim_{h\to 0}\frac{\arcsin(a+h)-\arcsin(a)}{h} = 2$, which of the following could be the value of a?

$(\arcsin a)' = \frac{1}{\sqrt{1-a^2}}$

$2 = \frac{1}{\sqrt{1-a^2}}$

$2\sqrt{1-a^2} = 1$

$\sqrt{1-a^2} = \frac{1}{2}$

$1-a^2 = \frac{1}{4}$

$-a^2 = -\frac{3}{4}$

$a^2 = \frac{3}{4}$

$a = \pm\frac{\sqrt{3}}{2}$

(A) $\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $\frac{1}{2}$

(E) 2

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APCLASSROOM MC 5

If $f(x) = \sin^{-1}x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{4}{7}$

(D) 2

$$\frac{df}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\left.\frac{df}{dx}\right|_{x=\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{1-\frac{3}{4}}}$$

$$= \frac{1}{\sqrt{\frac{1}{4}}}$$

$$= \frac{1}{\frac{1}{2}}$$

$$= 2$$

APCLASSROOM MC 6

$\frac{d}{dx}\left(\tan^{-1}x + 2\sqrt{x}\right) =$

$$\frac{1}{1+x^2} + 2\cdot\frac{1}{2}x^{-\frac{1}{2}}$$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

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APCLASSROOM MC 7

If $y = \arctan(\cos x)$, then $\frac{dy}{dx}$ = $\frac{1}{1+(\cos x)^2} \cdot (-\sin x)$

(A) $\frac{-\sin x}{1+\cos^2 x}$

(B) $-(\text{arcsec}(\cos x))^2 \sin x$

(C) $(\text{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$

(E) $\frac{1}{1+\cos^2 x}$

APCLASSROOM MC 14

derivative

What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2

(B) $\frac{1}{2}$

(C) 0

(D) $-\frac{1}{2}$

(E) -2

$y' = \frac{1}{1+(4x)^2} \cdot 4$

$y'\left(\frac{1}{4}\right) = \frac{4}{1+(4\cdot\frac{1}{4})^2}$

$= \frac{4}{2}$

$= 2$

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APCLASSROOM MC 15

POT

An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

(A) $x - 2y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

(E) $\pi x - 2y = 0$

SOT

$$y' = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2}$$

$$y'(0) = \frac{1}{1+\left(\frac{0}{2}\right)^2} \cdot \frac{1}{2}$$

$$= \frac{1}{1+0} \cdot \frac{1}{2}$$

$$= \frac{1}{2}$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$

$$2y = x$$

$$0 = x - 2y$$