

Unit 3.6 Calculating Higher-Order Derivatives

TOPIC QUESTION 1 CALCULATOR

A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

x	-1	0	1	2
$g(x)$	6	4	2	-1
$g'(x)$	-1	-7	-2	-3

The table above gives selected values for a differentiable and decreasing function g and its derivative. Let f be the function with $f(1) = 0$ and derivative given by $f'(x) = x \sin(x^2)$

- a. Find $f''(e^{1.5})$. Express your answer as a decimal approximation.

$$f''(e^{1.5}) \approx 14.144 \quad +1$$

- b. Let h be the function defined by $h(x) = f(g(3x))$. Find $h'(0)$. Express your answer as a decimal approximation.

$$h'(x) = f'(g(3x)) \cdot g'(3x) \cdot 3$$

Chain

$$\begin{aligned} h'(0) &= f'(g(0)) \cdot g'(0) \cdot 3 \\ &= f'(4) \cdot (-7) \cdot 3 \\ &= 4 \sin(16) \cdot (-21) \\ h'(0) &\approx -24.184 \text{ (or } -24.183) \end{aligned}$$

- c. Write an equation for the line tangent to the graph of g^{-1} , the inverse function of g , at $x = -1$.

$$\frac{g}{(2, -1), m = -3} \quad \frac{g^{-1}}{(-1, 2), m = -\frac{1}{3} + 1}$$

PUT SOT

$$y - 2 = -\frac{1}{3}(x + 1)$$

+1

- d. The point $(1, 4)$ lies on the curve in the xy -plane given by the equation $f(x) + (g(x))^2 = xy$. What is the value of $\frac{dy}{dx}$ at the point $(1, 4)$? Express your answer as a decimal approximation.

$$\begin{aligned} f'(x) + 2g(x) \cdot g'(x) &= 1 \cdot 4 + x \frac{dy}{dx} \quad +2 \\ \text{Chain} & \quad \text{Product} \\ @((1, 4)) \quad f'(1) + 2g(1) \cdot g'(1) &= 4 + 1 \cdot \frac{dy}{dx} \\ 1 \cdot \sin(1^2) + 2 \cdot (2) \cdot (-2) &= 4 + \frac{dy}{dx} \\ -8 & \\ \sin 1 - 12 &= \frac{dy}{dx} \\ \frac{dy}{dx} &= -11.154 \text{ (or } -11.158) \end{aligned}$$

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TOPIC QUESTION 3

Let f be the function given by $f(x) = 2x^3 - 5x^2 + 8x + 1$ on the closed interval $[0, 3]$. What is the instantaneous rate of change of f' at $x = 1$?

- a. 2
- b. 4
- c. 6
- d. 8

FIND $f''(1)$

$$f'(x) = 6x^2 - 10x + 8$$

$$f''(x) = 12x - 10$$

$$f''(1) = 12 \cdot 1 - 10$$

$$f''(1) = 2$$

TOPIC QUESTION 4

Let f be the function given by $f(x) = \sin x + e^{-x} + 3x$. Which of the following statements is true for $y = f(x)$?

- a. $y'' = \sin x + e^{-x}$
- b. $\frac{d^3y}{dx^3} = \frac{dy}{dx}$
- c. $f^{(4)}(x) = f'(x) \cdot f'''(x)$
- d. $y - \frac{d^4y}{dx^4} = 3x$

$$f' = \cos x - e^{-x} + 3$$

$$f'' = -\sin x + e^{-x}$$

$$f''' = -\cos x - e^{-x}$$

$$f^{(4)} = -\sin x + e^{-x}$$

TOPIC QUESTION 5

If $y = e^{x^3}$, then $\frac{d^2y}{dx^2} =$

- a. $18x^3e^{x^3}$
- b. $9x^4e^{2x^3}$
- c. $(6x + 3x^2)e^{x^3}$
- d. $(6x + 9x^4)e^{x^3}$

$$\begin{aligned} y &= 3x^2 e^{x^3} && \text{link} \cdot \text{link} \\ y' &= 6x \cdot e^{x^3} + 3x^2 \cdot (3x^2 e^{x^3}) && \text{product} \\ y'' &= \end{aligned}$$

$$y'' = e^{x^3} [6x + 9x^4]$$

AP CLASSROOM MC 1

If $y = \cos x - \ln(2x)$, then $\frac{d^3y}{dx^3} =$

- a. $\sin x - \frac{2}{x^3}$
- b. $-\sin x - \frac{2}{x^3}$
- c. $\sin x - \frac{1}{x^3}$
- d. $-\sin x - \frac{1}{x^3}$

$$y' = -\sin x - \frac{1}{2x} \cdot 2 = -\sin x - x^{-1}$$

$$y'' = -\cos x + x^{-2}$$

$$y''' = \sin x - 2x^{-3} = \sin x - \frac{2}{x^3}$$

AP CLASSROOM MC 2

If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4, 3)$?

- a. $-\frac{25}{27}$
- b. $-\frac{7}{27}$
- c. $\frac{7}{27}$
- d. $\frac{3}{4}$
- e. $\frac{25}{27}$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{-1 \cdot y - (-x) \cdot \frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y + x \left(-\frac{x}{y}\right)}{y^2}$$

$$\begin{aligned} \frac{d^2y}{dx^2} \Big|_{(4,3)} &= \frac{-3 + 4 \left(-\frac{4}{3}\right)}{3^2} \\ &= \frac{\left(-3 - \frac{16}{3}\right) \cdot 3}{9} \cdot 3 \\ &= \frac{-9 - 16}{27} \\ &= -\frac{25}{27} \end{aligned}$$