Unit 4 Progress Check MC Online

Scratch paper

Question 1

The mass of a colony of bacteria, in grams, is modeled by the function P given by $P(t) = 2 + 5\tan^{-1}\left(\frac{t}{2}\right)$, where t is measured in days. What is the instantaneous rate of change of the mass of the colony, in grams per day, at the moment the colony reaches a mass of 6 grams?

$6 = 2 + 5\tan^{-1}\left(\frac{t}{2}\right)$

$t = 2.05928$

$P'(2.05928) = 1.2135$

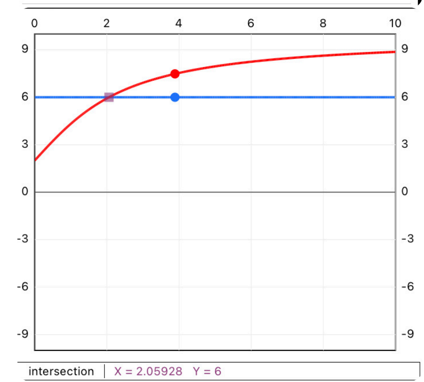
$\frac{d}{dX}(Y_2)_{X=2.05928}$

1.2135

Plot 1 Plot 2 Plot 3

$Y_1 = 6$

$Y_2 = 2 + 5\tan^{-1}(X/2)$



Ⓐ -0.606

Ⓑ 0.250

Ⓒ 1.214

Ⓓ 1.942

Question 2

SOT POT

The derivative of the function A is given by $A'(t) = 2 + 9e^{0.4\sin t}$ and $A(1.2) = 7.5$. If the linear approximation to $A(t)$ at $t = 1.2$ is used to estimate $A(t)$, at what value of t does the linear approximation estimate that $A(t) = 15$?

$y = 15$

$y - 7.5 = (2 + 9e^{0.4\sin t})(t - 1.2)$

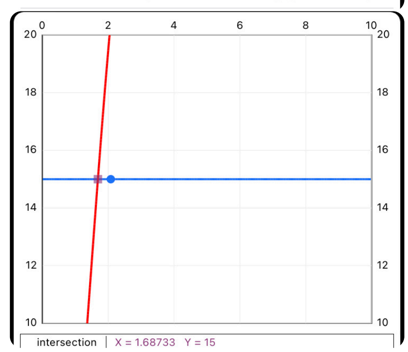
$y = 7.5 + (2 + 9e^{0.4\sin t})(t - 1.2)$

$15 = 7.5 + (2 + 9e^{0.4\sin t})(t - 1.2)$

Plot 1 Plot 2 Plot 3

$Y_1 = 15$

$Y_2 = 7.5 + (2 + 9e^{0.4\sin(X)})(X - 1.2)$



Ⓐ 0.498

Ⓑ 1.166

Ⓒ 1.698

Ⓓ 2.400

Question 3

Oil is spilled onto a kitchen floor. The area covered by the oil at time t is given by the function A, where $A(t)$ is measured in square centimeters and t is measured in seconds. Which of the following gives the rate at which the area covered by the oil is changing at time $t = 10$?

$A(t) = \text{Area cm}^2$

$t = \text{time seconds}$

$A'(10)$

Ⓐ $A(10)$

Ⓑ $A(11) - A(9)$

Ⓒ $\frac{A(10)}{10}$

Ⓓ $A'(10)$

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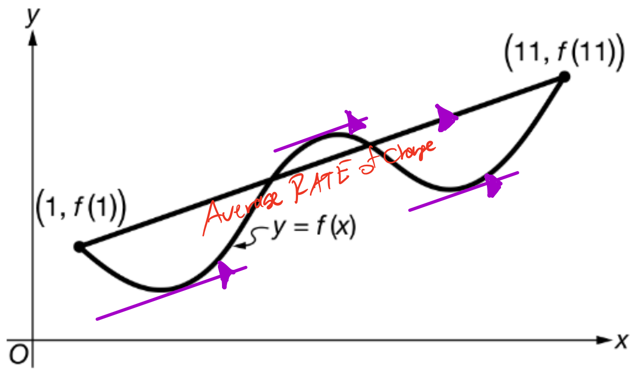
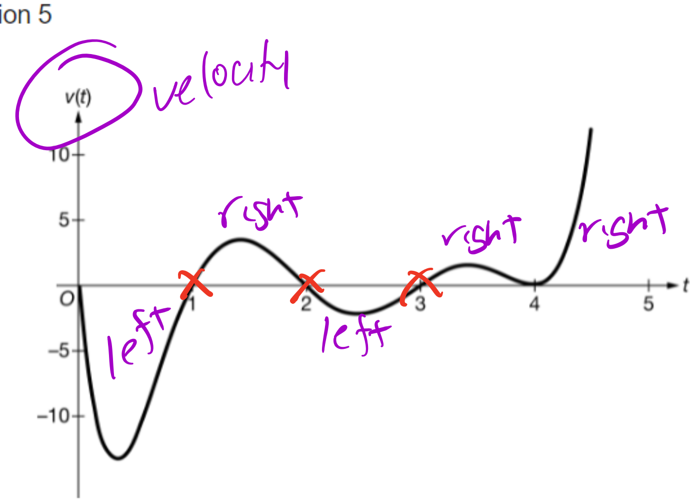
Question 4



The figure above shows the graph of the differentiable function f for $1 \leq x \leq 11$ and the secant line through the points $(1, f(1))$ and $(11, f(11))$. For how many values of x in the closed interval $[1, 11]$ does the instantaneous rate of change of f at x equal the average rate of change of f over that interval?

- (A) Zero
- (B) Two
- (C) Three
- (D) Four

Question 5



A particle moves along the y-axis. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 < t < 4.5$. How many times does the particle change direction over the time interval $0 < t < 4.5$?

- (A) Three
- (B) Four
- (C) Five
- (D) Eight

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Question 6

A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 + 3t^2 - 36t + 50$. What is the total distance traveled by the particle over the time interval $0 \leq t \leq 5$?

need to find distance left + right

Ⓐ 145

Ⓑ 180

Ⓒ 195

Ⓓ 233

① $x'(t) = 6t^2 + 6t - 36$

$0 = 6(t^2 + t - 6)$

$0 = 6(t+3)(t-2)$

② x': NEG on $(0, 2)$, POS on $(2, 5)$

③ x: left on $(0, 2)$, right on $(2, 5)$

④ $\text{total} = |x(0) - x(2)| + |x(5) - x(2)|$

⑤ Plot 1 Plot 2 Plot 3

$\backslash Y_1 = 2X^3 + 3X^2 - 36X + 50$

$|Y_1(0) - Y_1(2)| + |(Y_1(5) - Y_1(2))|$

233

Question 7

A particle moves along the y-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 6t^2 + 9t$. Over the time interval $0 < t < 4$, for what values of t is the speed of the particle increasing?

$v(t)$ and $a(t)$ are same signs.

Ⓐ $2 < t < 4$

Ⓑ $3 < t < 4$ only

Ⓒ $0 < t < 1$ and $3 < t < 4$

Ⓓ $1 < t < 2$ and $3 < t < 4$

$v = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t-1)$

v: POS on $(0, 1)$, NEG on $(1, 3)$, POS on $(3, 4)$

a: NEG on $(0, 2)$, POS on $(2, 4)$

$a(t) = 6t - 12$

$0 = 6(t-2)$

Question 8

Charles's law states that if the pressure of a dry gas is held constant, then the volume V of the gas and its temperature T, measured in degrees Kelvin, satisfy the relationship $V = cT$, where c is a constant. Which of the following best describes the relationship between the rate of change, with respect to time t, of the volume and the rate of change, with respect to time t, of the temperature?

Ⓐ $\frac{dV}{dt} = T\frac{dc}{dt}$

Ⓑ $\frac{dV}{dt} = c\frac{dT}{dt}$

Ⓒ $\frac{dV}{dT} = c$

Ⓓ $1 = c\frac{dT}{dV}$

$\frac{d}{dt}(V = cT)$

$\frac{dV}{dt} = c \cdot \frac{dT}{dt}$

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Question 9

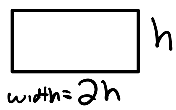
A rectangle has width w inches and height h inches, where the width is twice the height. Both w and h are functions of time t, measured in seconds. If A represents the area of the rectangle, which of the following gives the rate of change of A with respect to t?

(A) $\frac{dA}{dt} = 4h \text{ in/sec}$

(B) $\frac{dA}{dt} = 3h\frac{dh}{dt} \text{ in}^2\text{/sec}$

(C) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in/sec}$

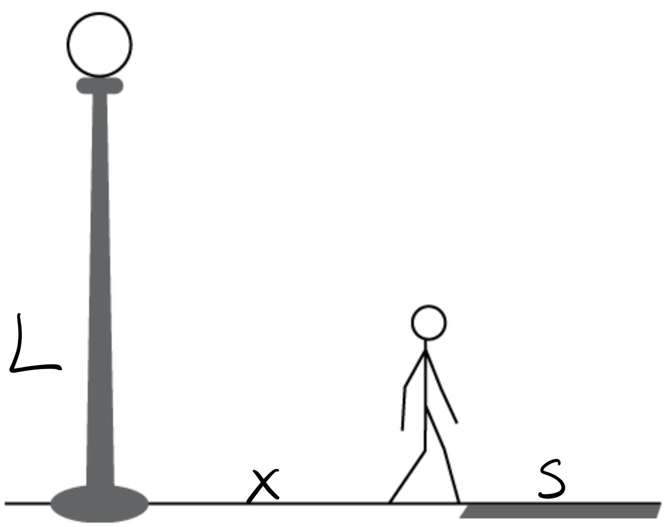
(D) $\frac{dA}{dt} = 4h\frac{dh}{dt} \text{ in}^2\text{/sec}$



$A = 2h^2 \text{ in}^2$

$\frac{dA}{dt} = 4h \cdot \frac{dh}{dt} \text{ in}^2\text{/sec}$

Question 10



A person whose height is M feet is walking away from the base of a streetlight along a straight road, as shown in the figure above. The height of the streetlight is L feet. At time t seconds, the person is x feet from the streetlight, and the length of the person's shadow is s feet. The quantities are related by the equation $\frac{1}{L}(x+s) = \frac{1}{M}s$, where L and M are constants. Which of the following best describes the relationship between the rate of change of x with respect to time and the rate of change of s with respect to time?

(A) $\frac{dx}{dt} = \frac{L}{M}s - s$

(B) $\frac{dx}{dt} = \frac{L}{M}s - \frac{ds}{dt}$

(C) $\frac{dx}{dt} = \frac{L}{M}\frac{ds}{dt} - s$

(D) $\frac{dx}{dt} = \frac{L}{M}\frac{ds}{dt} - \frac{ds}{dt}$

$\frac{1}{L}(x+s) = \frac{1}{M}s$

$\frac{d}{dt}\left(\frac{1}{L}x + \frac{1}{L}s = \frac{1}{M}s\right)$

$\frac{1}{L}\frac{dx}{dt} + \frac{1}{L}\frac{ds}{dt} = \frac{1}{M}\frac{ds}{dt}$

$\frac{dx}{dt} + \frac{ds}{dt} = \frac{L}{M}\frac{ds}{dt}$

$\frac{dx}{dt} = \frac{L}{M}\frac{ds}{dt} - \frac{ds}{dt}$

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Question 11

A particle moves on the circle $x^2 + y^2 = 25$ in the xy-plane for time $t \geq 0$. At the time when the particle is at the point $(3, 4)$, $\frac{dx}{dt} = 6$. What is the value of $\frac{dy}{dt}$ at this time?

(A) $\frac{dy}{dt} = \frac{19}{8}$

(B) $\frac{dy}{dt} = -\frac{3}{4}$

(C) $\frac{dy}{dt} = -\frac{11}{8}$

(D) $\frac{dy}{dt} = -\frac{9}{2}$

$x^2+y^2=25$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

FIND $\frac{dy}{dt}$

$x=3$

$y=4$

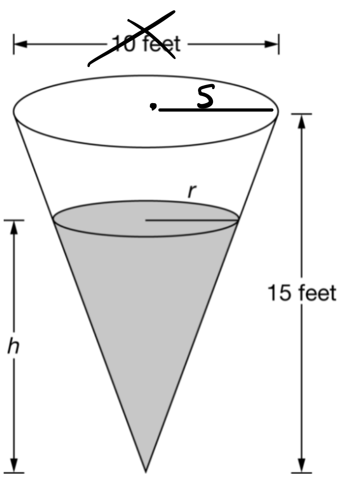
$\frac{dx}{dt}=6$

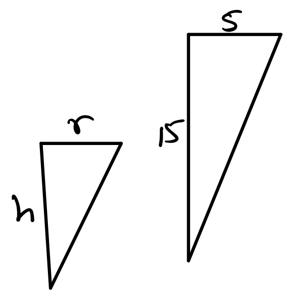
$2(3)6+2(4)\frac{dy}{dt}=0$

$\frac{dy}{dt}=\frac{-36}{8}$

$\frac{dy}{dt}=\frac{-9}{2}$

Question 12





$\frac{r}{h}=\frac{5}{15}$

$r=\frac{1}{3}h$

A water tank is in the shape of a right circular cone as shown above. The diameter of the cone is 10 feet, and the height is 15 feet. The shape of the water in the tank is conical with radius r feet and height h feet. At noon, water is leaking from the bottom of the tank at a rate of 12 cubic feet per hour, and the volume of water in the tank is 27π cubic feet. At noon, what is the rate at which the height of the water in the tank is changing? (The volume V of a right circular cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(A) Decreasing at $\frac{4}{9\pi}$ feet per hour

(B) Increasing at $\frac{4}{9\pi}$ feet per hour

(C) Increasing at $\frac{4}{3\pi}$ feet per hour

(D) Decreasing at $\frac{4}{3\pi}$ feet per hour

$\frac{h^3}{27}\pi=27\pi$

$V=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2h$

$V=\frac{1}{27}\pi h^3$

$\frac{dV}{dt}=\frac{1}{9}\pi h^2\cdot\frac{dh}{dt}$

Noon FIND $\frac{dh}{dt}$

$\frac{dV}{dt}=-12\ ft^3/hr$

$V=27\pi\ ft^3$

what is h?

$27\pi=\frac{1}{27}\pi h^3$

$729=h^3$

$9=h$

$\frac{dV}{dt}=\frac{1}{9}\pi h^2\cdot\frac{dh}{dt}$

$-12=\frac{1}{9}\pi(9)^2\cdot\frac{dh}{dt}$

$-12=9\pi\cdot\frac{dh}{dt}$

$\frac{-4}{3\pi}=\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{-4}{3\pi}\ ft/hr$

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Question 13

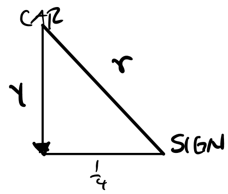
Two straight roads intersect at right angles. A car is traveling south toward the intersection while a radar-monitored-speed sign is positioned $\frac{1}{4}$ mile east of the intersection. The speed sign provides both the distance from the sign to an approaching car and the rate at which the distance between the sign and the car is changing. At a certain time, the speed sign shows that the approaching car is exactly $\frac{1}{2}$ mile away from the sign and that the rate at which the distance between the car and the sign is decreasing is 60 miles per hour. Which of the following is true about the distance between the car and the intersection at this instant?

(A) ~~The distance is increasing at a rate of $\frac{120}{\sqrt{3}}$ miles per hour.~~

(B) The distance is decreasing at a rate of $\frac{120}{\sqrt{3}}$ miles per hour.

(C) ~~The distance is increasing at a rate of $\frac{30}{\sqrt{3}}$ miles per hour.~~

(D) The distance is decreasing at a rate of $\frac{30}{\sqrt{3}}$ miles per hour.



$(\frac{1}{4})^2 + y^2 = r^2$

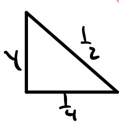
$2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

L

$r = \frac{1}{2}$

$\frac{dr}{dt} = 60$ mi/hr

FIND $\frac{dy}{dt}$



$(\frac{1}{4})^2 + y^2 = (\frac{1}{2})^2$

$\frac{1}{16} + y^2 = \frac{1}{4}$

$1 + 16y^2 = 4$

$16y^2 = 3$

$y^2 = \frac{3}{16}$

$y = \frac{\sqrt{3}}{4}$

$2(\frac{\sqrt{3}}{4})\frac{dy}{dt} = 2(\frac{1}{2})60$

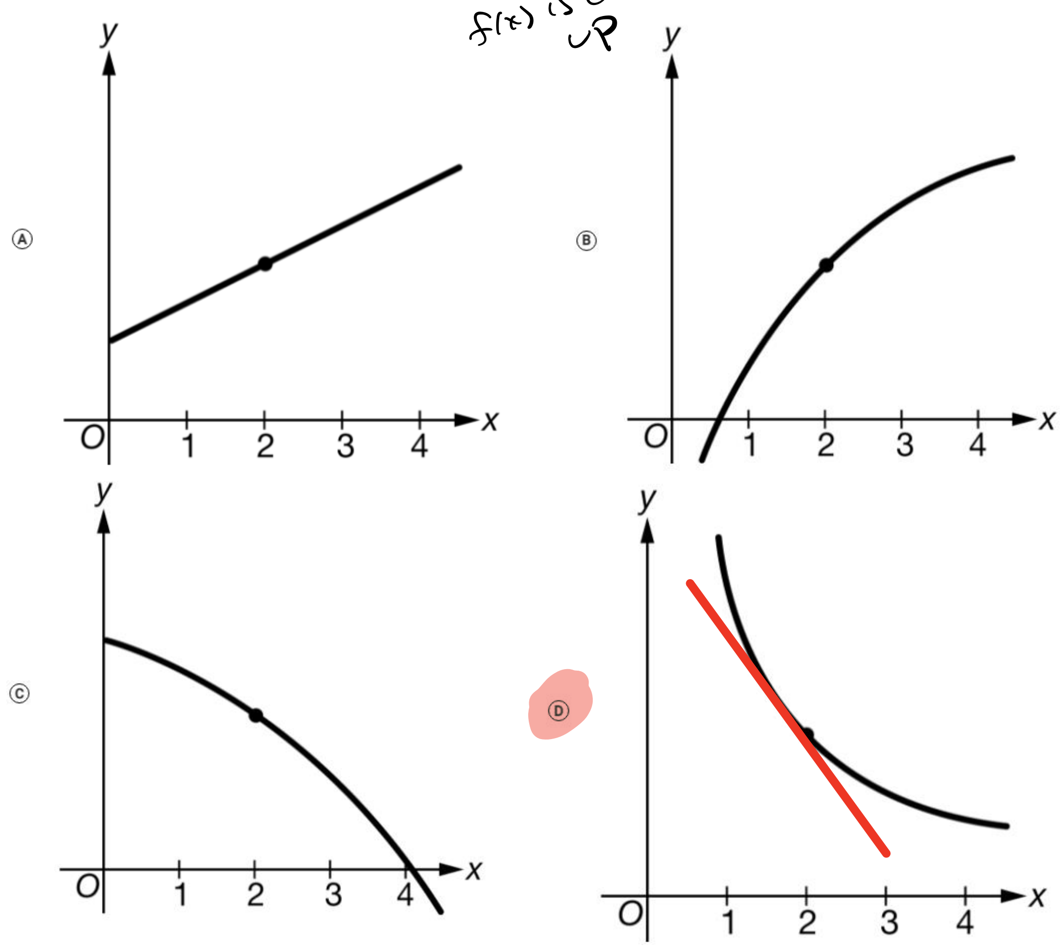
$\frac{dy}{dt} = 60 \cdot \frac{2}{\sqrt{3}}$

$\frac{dy}{dt} = \frac{120}{\sqrt{3}}$ mi/hr

Question 14

The locally linear approximation of the differentiable function f at $x = 2$ is used to approximate the value of $f(2.3)$. The approximation at $x = 2.3$ is an underestimate of the corresponding function value at $x = 2.3$. Which of the following could be the graph of f?

$f(x)$ is concave up



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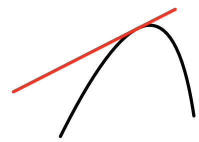
Question 15

The line tangent to the graph of the twice-differentiable function f at the point $x = 5$ is used to approximate the value of $f(5.25)$. Which of the following statements guarantees that the tangent line approximation at $x = 5.25$ is an overestimate of $f(5.25)$? Concave down



(A) The function f is decreasing on the interval $5 \leq x \leq 5.25$.

(B) The function f is increasing on the interval $5 \leq x \leq 5.25$.

(C) The graph of the function f is concave down on the interval $5 \leq x \leq 5.25$.

(D) The graph of the function f is concave up on the interval $5 \leq x \leq 5.25$.

Question 16

Which of the following limits does not yield an indeterminate form?

(A) $\lim\limits_{x \to 0} \frac{3x^2}{x - \sin x}$ $\qquad \frac{0}{0}$

(B) $\lim\limits_{x \to 2} \frac{\ln\left(\frac{x}{2}\right)}{x^2 - 5x + 6}$ $\qquad \frac{0}{0}$

(C) $\lim\limits_{x \to \pi} \frac{x - \pi}{\cos x}$ $\qquad \frac{0}{-1}$

(D) $\lim\limits_{x \to \infty} \frac{e^{3x}}{x^{100}}$ $\qquad \frac{\infty}{\infty}$

Question 17

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
3	2	-4	1	2

Selected values of the twice-differentiable functions f and g and their derivatives are given in the table above. The value of $\lim\limits_{x \to 3} \frac{x^3 f(x) - 54}{g(x) - 1}$ is

$$= \lim_{x \to 3} \frac{3x^2 \cdot f(x) + x^3 \cdot f'(x)}{g'(x)} = \frac{3(3)^2 \cdot (2) + 3^3 \cdot (-4)}{2} = \frac{54 + 27(-4)}{2} = 27 + 27(-2) = -27$$

L'HOSPITALS

$$\left.\begin{array}{l} \lim\limits_{x \to 3}\left(x^3 f(x) - 54\right) = 27 \cdot (2) - 54 = 0 \\ \lim\limits_{x \to 3}\left(g(x) - 1\right) = 1 - 1 = 0 \end{array}\right\} \frac{0}{0}$$

(A) -108

(B) -54

(C) -27

(D) nonexistent

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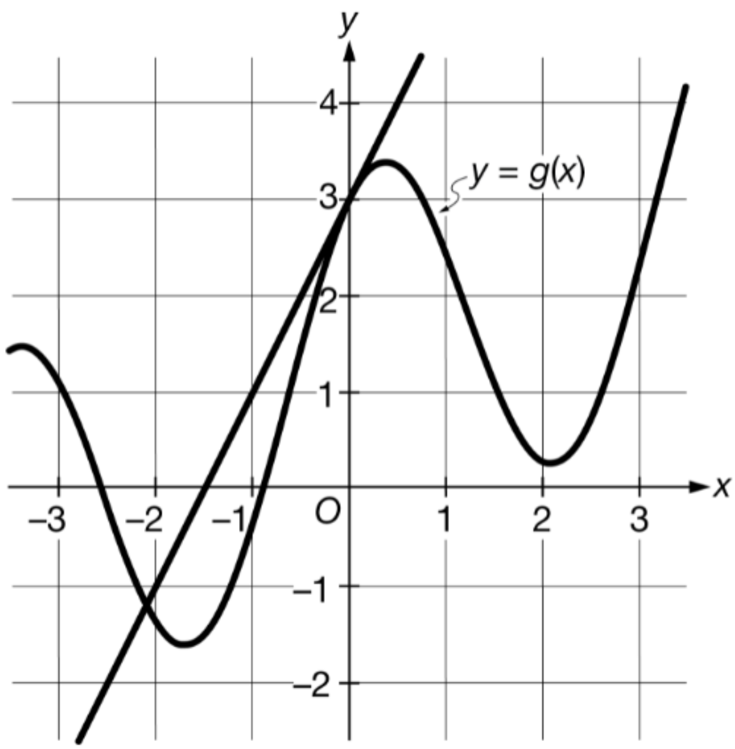
Question 18



The figure above shows the graph of the twice-differentiable function g and the line tangent to the graph of g at the point $(0,3)$. The value of $\lim_{x\to 0}\frac{g(x)e^{-x}-3}{x^2-2x}$ is

$$\lim_{x\to 0}\frac{g'(x)e^{-x}+g(x)e^{-x}(-1)}{2x-2} = \frac{g'(0)e^{-0}+g(0)e^{-0}(-1)}{2(0)-2}$$

L' HOSPITALS

$$= \frac{2\cdot 1+3\cdot 1(-1)}{-2}$$

$$= \frac{-1}{-2}$$

$$= \frac{1}{2}$$

$$\left.\begin{array}{l}\lim_{x\to 0}(g(x)e^{-x}-3)=0\\ \lim_{x\to 0}(x^2-2x)=0\end{array}\right\}\ \frac{0}{0}$$

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{5}{2}$

(D) nonexistent