Name: ______________________________ Date: _______________ Period: _____ **Review**

End of Unit 1 Review– Limits and Continuity

Lessons 1.10 through 1.16.

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 1 (including the Mid-Unit Review).

1. If $f(x) = \frac{x+3}{x^2-2x-15}$, identify the type of each discontinuity and where it is located.

$f(x) = \frac{x+3}{(x-5)(x+3)}$

point discontinuity @ $x=-3$

infinite discontinuity @ $x=5$

State whether the function is continuous at the given x values. Justify your answers!

2. $f(x) = \begin{cases} \cos(3x), & x < 0 \\ \tan x, & 0 \le x < \frac{\pi}{4} \\ \sin(2x), & x \ge \frac{\pi}{4} \end{cases}$

Continuous at $x = 0$?

I $f(0) = \tan(0) = 0$

II $\lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x)$

$\cos(3\cdot 0) \ne \tan(0)$

$1 \ne 0$

$f(x)$ is not continuous at $x=0$

Continuous at $x = \frac{\pi}{4}$?

I $f\left(\frac{\pi}{4}\right) = \sin\left(2\cdot\frac{\pi}{4}\right) = 1$

II $\lim_{x\to \frac{\pi}{4}^-} f(x) = \lim_{x\to \frac{\pi}{4}^+} f(x)$

$\tan\left(\frac{\pi}{4}\right) = \sin\left(2\cdot\frac{\pi}{4}\right)$

$1 = 1$

III $f\left(\frac{\pi}{4}\right) = \lim_{x\to\frac{\pi}{4}} f(x)$

$\therefore f$ is continuous at $x = \pi/4$

Find the domain of each function.

3. $h(t) = \frac{\sqrt{t+3}}{t-5}$

Denom $\ne 0$

$t \ne 5$

radicand ≥ 0

$t+3 \ge 0$

$t \ge -3$

$[-3, 5) \cup (5, \infty)$

4. $f(x) = \ln\left(\frac{2}{x-1}\right)$

Denom $\ne 0$

$x-1 \ne 0$

$x \ne 1$

Arguments > 0

$\frac{2}{x-1} > 0$

$x-1 > 0$

$x > 1$

$(1, \infty)$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2+6x+8}{x+2}$ when $x \ne -2$, then $f(-2) =$ 2

↳ what is y-coordinate of hole?

$f(x) = \frac{\cancel{(x+2)}(x+4)}{\cancel{x+2}}$

$f(x) = x+4$

Hole @ $(-2, 2)$

$f(-2) = -2+4$

$f(-2) = 2$

6. Let f be the function defined by $f(x) = \begin{cases} \frac{x^2+8x+12}{x+6}, & x \ne -6 \\ b, & x = -6 \end{cases}$. For what value of b is f continuous at $x = -6$?

↳ what is y-coordinate of hole?

$f(x) = \frac{(x+6)(x+2)}{x+6} = x+2$, hole $(-6, -4)$

$y = -6+2$

$y = -4$

$b = -4$

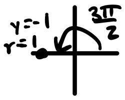
Evaluate the limit.

7. $\lim_{x\to\infty}\sin\left(\frac{x+3\pi x^2}{2x^2}\right)$

growing at same rate $\therefore$ HA @ $y=\frac{a}{b}$

where is HA?

$= \lim_{x\to\infty}\sin\left(\frac{3\pi}{2}\right)$



$= -1$

8. $\lim_{x\to-5^-}\frac{-3}{25-x^2} = \frac{-3}{(5-x)(5+x)}$

HA @ $x=-5$, but is graph going up or down as it approaches -5^-?

$x\to-5^-$	$\frac{-3}{(5-x)(5+x)}$
-5.0001	$\frac{-3}{(5--5.0001)(5-5.0001)}$ $=\frac{NEG}{POS\cdot NEG}$ $= POS$

$\therefore \lim_{x\to-5^-}\frac{-3}{25-x^2} = \infty$

9. $\lim_{x\to\infty}\frac{\sin x}{x} = 0$

Bottom grows faster. $\therefore$ HA @ $y=0$

where is HA?

10. $\lim_{x\to\infty}\frac{4x^5-2x^2+3}{3x^2+2x^5-x^4}$

growing at same rate $\therefore$ HA @ $y=\frac{a}{b}$

where is HA?

$=\frac{4}{2}$

$=2$

11. $\lim_{x\to-1}\frac{x^2+1}{x+1}$ DNE

HA @ $x=-1$, but is graph going up or down as it approaches -1?

$x\to-1^-$	$\frac{x^2+1}{x+1}$
-1.001	$\frac{(-1.001)^2+1}{-1.001+1}$ $=\frac{POS}{NEG}$ $-\infty$

$x\to-1^+$	$\frac{x^2+1}{x+1}$
1.001	$\frac{(1.001)^2+1}{1.001+1}$ $=\frac{POS}{POS}$ $=\infty$

12. $\lim_{x\to\infty}x^5 3^{-x} = \lim_{x\to\infty}\frac{x^5}{3^x} = 0$

Bottom grows faster. $\therefore$ HA @ $y=0$

where is HA?

13. Identify all horizontal asymptotes of $f(x)=\frac{\sqrt{16x^6+x^3+5x}}{5x^3-8x}$.

$\lim_{x\to\infty}\frac{\sqrt{16x^6}}{5x^3} = \frac{4}{5}$ growing at same rate $\therefore$ HA @ $y=\frac{a}{b}$

$\lim_{x\to-\infty}\frac{\sqrt{16x^6}}{5x^3} = -\frac{4}{5}$ growing at same rate $\therefore$ HA @ $y=\frac{a}{b}$

$\therefore$ HA @ $y=\frac{4}{5}$, $y=-\frac{4}{5}$

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Unit 2 Review – Differentiation: Definition & Fundamental Properties

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you should review all packets from Unit 2.

Find the average rate of change of each function on the given interval. Use appropriate units if necessary.

1. $w(x) = \ln x; \quad 1 \le x \le 7$

$ARC = \frac{\ln(1) - \ln 7}{1-7} = \frac{0 - \ln 7}{-6} = \frac{\ln 7}{6}$

2. $s(t) = -t^2 - t + 4; \; [1, 5]$
   t represents seconds
   s represents feet

$ARC = \frac{s(1) - s(5)}{1-5} = \frac{2-(-26)}{-4} = \frac{28}{-4} = -7 \text{ ft/sec}$

3. Find the derivative of $y = 2x^2 + 3x - 1$ by using the definition of the derivative. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{[2(x+h)^2 + 3(x+h) - 1] - [2x^2 + 3x - 1]}{h} = \lim_{h\to 0} \frac{2(x^2+2hx+h^2) + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$

$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 + \cancel{3x} + 3h \cancel{-1} \cancel{-2x^2} \cancel{-3x} \cancel{+1}}{h} = \lim_{h\to 0} \frac{h(4x+2h+3)}{h}$

$= \lim_{h\to 0} (4x + 2h + 3) = 4x + 2(0) + 3 = 4x + 3$

4. For the function $h(t)$, h is the temperature of the oven in Fahrenheit, and t is the time measured in minutes.
   a. Explain the meaning of the equation $h(\underset{\text{min}}{15}) = 420\,°F$ AT 15 minutes, the oven's temp is 420°F

   b. Explain the meaning of the equation $h'(43) = -11\,°F/\text{min}$

   AT 43 minutes, the oven's temp is decreasing by 11°F per minute

Find the derivative of each function.

5. $f(x) = 4 - \frac{1}{2x^2} = 4 - \frac{1}{2}x^{-2}$

$f'(x) = +x^{-3}$

$f'(x) = \frac{1}{x^3}$

6. $g(x) = 3\sqrt{x} - \frac{6}{x^2} + 5\pi^3$

$g(x) = 3x^{1/2} - 6x^{-2} + 5\pi^3$

$\frac{dg}{dx} = \frac{3}{2}x^{-1/2} + 12x^{-3}$

$\frac{dg}{dx} = \frac{3}{2\sqrt{x}} + \frac{12}{x^3}$

7. $h(x) = 4e^x - 2\cos x$

$h'(x) = 4e^x + 2\sin x$

8. $s(t) = t^2 \sin(t)$

$s'(t) = 2t \cdot \sin(t) + t^2 \cos(t)$

9. $d(t) = 3\sqrt{t} \ln t$

$d'(t) = \frac{3}{2}t^{-\frac{1}{2}} \cdot \ln t + 3\sqrt{t} \cdot \frac{1}{t}$

$d'(t) = \frac{3 \ln t}{2\sqrt{t}} + \frac{3}{\sqrt{t}}$

10. $y = \frac{4}{x} - \sec x = 4x^{-1}$

$y' = -4x^{-2} - \sec(x)\tan(x)$

$y' = \frac{-4}{x^2} - \sec(x)\tan(x)$

11. $h(x) = \frac{2-x}{x+2}$

$h' = \frac{-1\cdot(x+2) - (2-x)\cdot 1}{(x+2)^2}$

$h' = \frac{-x-2-2+x}{(x+2)^2}$

$h' = \frac{-4}{(x+2)^2}$

Find the equation of the tangent line of the function at the given *x*-value.

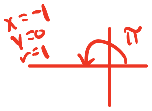
12. $f(x) = -2x^3 + 3x$ at $x = -1$.

PoT $(-1,-1)$	SoT $= -3$	Tangent
$f(-1) = -1$	$f' = -6x^2 + 3$	$y + 1 = -3(x+1)$
	$f'(-1) = -6(-1)^2 + 3$	
	$f'(-1) = -3$	

13. $f(x) = 4\sin x - 2$ at $x = \pi$

PoT $(\pi, -2)$	SoT $= -4$	tangent
$f(\pi) = -2$	$f' = 4\cos x$	$y + 2 = -4(x - \pi)$
	$f'(\pi) = -4$	

$x = -1$
$y = 0$
$r = 1$



14. Find the equation for the normal line of $y = \frac{1}{2}x^2 + \frac{3}{4}x - 4$ at $x = -3$

PoN $(-3, -\frac{7}{4})$	SoT	SoN $= \frac{4}{9}$	Normal
$y(-3) = \frac{1}{2}(-3)^2 + \frac{3}{4}(-3) - 4$	$y' = x + \frac{3}{4}$		$y + \frac{7}{4} = \frac{4}{9}(x+3)$
$= \frac{9}{2} - \frac{9}{4} - 4$	$y'(-3) = \frac{-12}{4} + \frac{3}{4}$		
$= \frac{18}{4} - \frac{9}{4} - \frac{16}{4}$	$y'(-3) = -\frac{9}{4}$		
$y(-3) = -\frac{7}{4}$			

15. If $f(x) = 3\sin x - 2e^x$ find $f'(0)$. No calculator!

$f'(x) = 3\cos x - 2e^x$

$f'(0) = 3\cos(0) - 2e^0$

$= 3\cdot 1 - 2\cdot 1$

$= 1$

A calculator is allowed on the following problems.

16. If $f(x) = x\sin(3x^2 - 2)$; find $f'(7)$.

$f'(7) = 260.246$

$\frac{d}{dX}(X\sin(3X^2-2))_{X=7}$

17. If $f(x) = \csc(3x)$ at $x = 2$.

$f'(2) \approx -36.899$

$\frac{d}{dX}(1/\sin(3X))_{X=2}$

18. Use the table below to estimate the value of $d'(120)$. Indicate units of measures.

t seconds	2	13	60	180	500
$d(t)$ feet	10	81	412	808	2,105

$d'(120) \approx \frac{d(60) - d(180)}{60 - 180} \approx \frac{412 - 808}{60 - 180}$ ft/sec is $f(x)$ continuous at $x=2$? AND does $\lim_{x\to 2} f'(x)$ exist?

19. Is the function differentiable at $x = 2$? Yes

$$f(x) = \begin{cases} 3x - 3x^2 - 5, & x < 2 \\ 7 - 9x, & x \geq 2 \end{cases}$$

I $f(2) = 7 - 9(2) = -11$

II $\lim_{x\to 2^-} f(x) = 3(2) - 3(2)^2 - 5 = -11$

$\lim_{x\to 2^+} f(x) = 7 - 9(2) = -11$

$\therefore \lim_{x\to 2} f(x)$ exists

III $f(2) = -11 = \lim_{x\to 2} f(x)$

$\therefore f$ is continuous at $x=2$

$\lim_{x\to 2^-} f' = 3 - 6(2) = -9$

$\lim_{x\to 2^+} f' = -9$

$\therefore \lim_{x\to 2} f'$ exists

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20. What values of a and b would make the function differentiable at $x = 4$?

$$f(x) = \begin{cases} a\sqrt{x} + bx^2 - 1, & x < 4 \\ \dfrac{16}{x} + bx, & x \geq 4 \end{cases}$$

① $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

$a\sqrt{4} + b(4)^2 - 1 = \frac{16}{4} + b\cdot 4$

$2a + 16b - 1 = 4 + 4b$

$2a - 5 = -12b$

② $\lim_{x\to 4^-} f'(x) = \lim_{x\to 4^+} f'(x)$

$\lim_{x\to 4^-} \left(\frac{1}{2}ax^{-\frac{1}{2}} + 2bx\right) = \lim_{x\to 4^+} \left(-16x^{-2} + b\right)$

$\frac{a}{2\sqrt{4}} + 2b(4) = \frac{-16}{4^2} + b$

$\frac{a}{4} + 8b = -1 + b$

$\frac{a}{4} = -1 - 7b$

$a = -4 - 28b$

③ $2(-4 - 28b) - 5 = -12b$

$-8 - 56b - 5 = -12b$

$-13 = 44b$

$\frac{-13}{44} = b$

④ $a = -4 - 28\left(\frac{-13}{44}\right)$

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Review

Unit 3 REVIEW – Composite, Implicit, and Inverse Functions

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 3.

Find the derivative.

1. $h(x) = \cos^2(4x) = [\cos(4x)]^2$

LINK LINK LINK

$h'(x) = 2[\cos(4x)] \cdot (-\sin(4x)) \cdot 4$

LINK LINK LINK

$h'(x) = -8\cos(4x)\sin(4x)$

2. $y = \ln\sqrt{x+3} = \ln[(x+3)^{\frac{1}{2}}]$

$y' = \frac{1}{(x+3)^{1/2}} \cdot \frac{1}{2}(x+3)^{-\frac{1}{2}}(1)$

$y' = \frac{1}{2(x+3)^{1/2}\cdot(x+3)^{1/2}}$

$y' = \frac{1}{2(x+3)}$

3. $x^2 + 2y^5 = 10xy$ Assume with respect to x?

$2x + 10y^4\frac{dy}{dx} = 10\cdot y + 10x\cdot\frac{dy}{dx}$

$10y^4\frac{dy}{dx} - 10x\cdot\frac{dy}{dx} = 10y - 2x$

$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$

$\frac{dy}{dx} = \frac{10y - 2x}{10y^4 - 10x}$

4. $y = \csc^{-1}(x^3)$

$\frac{dy}{dx} = \frac{1}{|x^3|\sqrt{(x^3)^2-1}}\cdot(3x^2)$

$= \frac{3}{|x|\sqrt{x^6-1}}$

For each problem, let f and g be differentiable functions where $g(x) = f^{-1}(x)$ for all x.

5. $f(6) = -1$, $f(4) = -2$, $f'(6) = 3$, and $f'(4) = 7$. What is the value of $g'(-1)$?

$f(x)$	$f^{-1}(x) = g(x)$
$(6,-1)$, $m=3$	$(-1,6)$, $m=\frac{1}{3}$

6. Let f be the function defined by $f(x) = x^3 + 3x + 1$. Let $g(x) = f^{-1}(x)$, where $g(-3) = -1$. What is the value of $g'(-3)$?

f	$f^{-1} = g$
② $(-1,-3)$, $m=6$	① $(-3,-1)$, $m=\frac{1}{6}$ ④

③

$f'(x) = 3x^2+3$

$f'(-1) = 3(-1)^2+3$

$= 3+3$

$= 6$

Find $\frac{d^2y}{dx^2}$ based on the given information.

7. $y = x^5 - e^{4x}$

$$\frac{dy}{dx} = 5x^4 - e^{4x}(4)$$

$$\frac{d^2y}{dx^2} = 20x^3 - 4e^{4x}(4)$$

8. $y = y^2 + x$

$$\frac{dy}{dx} = 2y\frac{dy}{dx} + 1$$

$$\frac{dy}{dx} - 2y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(1-2y) = 1$$

$$\frac{dy}{dx} = \frac{1}{1-2y} = (1-2y)^{-1}$$

$$\frac{d^2y}{dx^2} = -1(1-2y)^{-2}\left(-2\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{2}{(1-2y)^2} \cdot \frac{1}{1-2y}$$

$$\frac{d^2y}{dx^2} = \frac{2}{(1-2y)^3}$$

9. Find the equation of the tangent line.
$x^2 + 7y^2 = 8y^3$ at $(-6, 2)$

$$2x + 14y \cdot \frac{dy}{dx} = 24y^2\frac{dy}{dx}$$

$$2(-6) + 14(2)\frac{dy}{dx} = 24(2)^2\frac{dy}{dx}$$

$$-12 + 28\frac{dy}{dx} = 96 \cdot \frac{dy}{dx}$$

$$-12 = 68\frac{dy}{dx}$$

$$\frac{-12}{68} = \frac{dy}{dx}$$

$$y - 2 = \frac{-12}{68}(x+6)$$

10. If $x = y^2 - \cos x$ find $\frac{d^2y}{dx^2}$ at $\left(\frac{\pi}{6}, \frac{1}{2}\right)$.

$$1 = 2y\frac{dy}{dx} + \sin x$$

$$1 - \sin x = 2y\frac{dy}{dx}$$

$$\frac{1-\sin x}{2y} = \frac{dy}{dx} \xRightarrow{@\left(\frac{\pi}{6},\frac{1}{2}\right)} \frac{1-\sin\left(\frac{\pi}{6}\right)}{2\left(\frac{1}{2}\right)} = \frac{1-\frac{1}{2}}{1} = \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{(0-\cos x)\cdot 2y - (1-\sin x)\cdot 2\frac{dy}{dx}}{(2y)^2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\left(\frac{\pi}{6},\frac{1}{2}\right)} = \frac{-\cos\left(\frac{\pi}{6}\right)\cdot 2\left(\frac{1}{2}\right) - \left(1-\sin\frac{\pi}{6}\right)\cdot 2\cdot\left(\frac{1}{2}\right)}{\left[2\left(\frac{1}{2}\right)\right]^2}$$

$$= \frac{-\frac{\sqrt{3}}{2}\cdot 1 - \left(1-\frac{1}{2}\right)}{1^2}$$

$$= -\frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\left(\frac{\pi}{6},\frac{1}{2}\right)} = \frac{-\sqrt{3}-1}{2}$$

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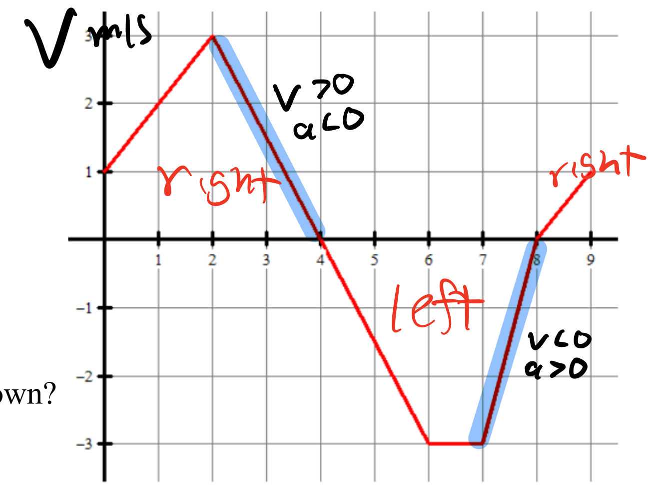
Unit 4 REVIEW – Contextual Application of Differentiation

Reviews do NOT cover all material from the lessons but should remind you of key points. To be prepared, you should review all packets from Unit 4.

1. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.



a) When does the body reverse direction?

$t=4$ and $t=8$

b) When is the body moving at a constant speed?

$6<t<7$

c) What is the body's maximum speed?

3 m/s

d) At what time interval(s) is the body slowing down?

a + v are opposite signs

$2<t<4$ and $7<t<8$

Find the following. Use L'Hospital's when possible.

2. $\lim\limits_{x\to 2} \frac{x-2}{x^2-7x+10}$ Produces $\frac{0}{0}$

L'Hospital's

$= \lim\limits_{x\to 2} \frac{1}{2x-7} = \frac{1}{2\cdot 2-7} = \frac{1}{-3}$

3. $\lim\limits_{x\to 0} \frac{3x^2}{e^x-1-x}$ produces $\frac{0}{0}$

L'Hospital's

$\lim\limits_{x\to 0} \frac{6x}{e^x-1}$ produces $\frac{0}{0}$

$\lim\limits_{x\to 0} \frac{6}{e^x} = \frac{6}{e^0} = 6$

4. $\frac{d}{dx}\frac{3x-2}{5x+1} = \frac{3(5x+1)-(3x-2)(5)}{(5x+1)^2}$

Quotient

$= \frac{15x+3-15x+10}{(5x+1)^2}$

$= \frac{13}{(5x+1)^2}$

5. If the length l of a rectangle is decreasing (NEG) at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?

$\frac{dl}{dt} = -2 \text{ in/min}$

$\frac{dw}{dt} = 2 \text{ in/min}$

$A = lw$

$\frac{dA}{dt} = \frac{dl}{dt}\cdot w + l\cdot\frac{dw}{dt}$

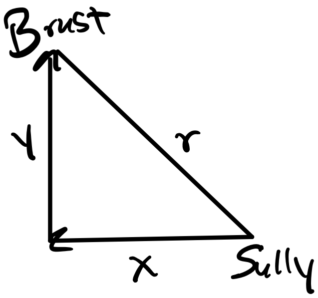
$\frac{dA}{dt} = -2w + l\cdot 2 = 2(l-w)$

(A) A is always increasing. (B) A is always decreasing. (C) A is increasing only when $l > w$.

(D) A is increasing only when $l < w$. (E) A remains constant.

The following problems are calculator active.

6. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?



$\frac{dy}{dt} = 15$ mi/hr

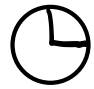
$\frac{dx}{dt} = -30$ mi/hr

$x^2 + y^2 = r^2$

$2x \cdot \frac{dx}{dt} + 2y \frac{dy}{dx} = 2r \cdot \frac{dr}{dt}$

$2x \cdot (-30) + 2y(15) = 2r \frac{dr}{dt}$

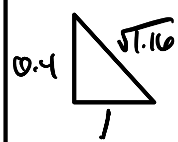
$-30x + 15y = r \cdot \frac{dr}{dt}$



$y = 0.4$

$x = 1$

$r = \sqrt{1.16}$



FIND $\frac{dr}{dt}$

$-30(1) + 15(0.4) = \sqrt{1.16} \frac{dr}{dt}$

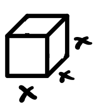
$-30 + 6 = \sqrt{1.16}$

$\frac{-24}{\sqrt{1.16}} = \frac{dr}{dt}$

The distance is decreasing by $\frac{24}{\sqrt{1.16}}$ mi/hr

HARD

7. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area S, what is the rate of change of the volume of the cube, in cubic centimeters per second?



$\frac{dx}{dt} = 0.2$ cm/sec

$S = 6x^2$

FIND $\frac{dV}{dt}$

$V = x^3$

$\frac{dV}{dx} = 3x^2 \frac{dx}{dt} = 3x^2(0.2)$

$= 3x^2 \cdot 2(0.1)$

$= 6x^2(0.1)$

$= S \cdot (0.1)$

(A) $0.1S$ (B) $0.2S$ (C) $0.6S$

(D) $0.04S$ (E) $0.008S$

8. The function $f(x) = (1 - \sin x)^2$ is concave up at $x = \frac{\pi}{6}$?

a. What is the estimate for $f(0.5)$ using the local linear approximation for f at $x = \frac{\pi}{6}$?

POT	SOT
$f(\frac{\pi}{6}) = \frac{1}{4}$	$f'(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$

$y - \frac{1}{4} = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$y = -\frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{4}$

$f(0.5) = -\frac{\sqrt{3}}{2}\left(0.5 - \frac{\pi}{6}\right) + \frac{1}{4}$

$f(0.5) \approx 0.270$

b. Is it an underestimate or overestimate? Explain.

f(x) is concave up ∴ f(0.5) is an underexstimate

Name: _______________________________ Date:_______________ **Corrective Assignment**

End of Unit 5 CA – Analytical Applications of Differentiation

1. **Calculator active problem.** The first derivative of the function f is given by
$$f'(x) = -2 + x + 3e^{-\cos(4x)}$$
How many points of inflection does the graph of f have on the interval $0 < x < \pi$?

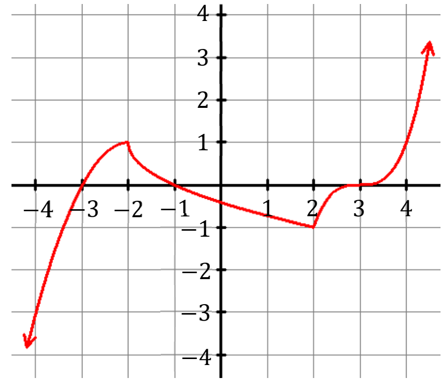
2. **Calculator active problem.** The rate of money in a particular mutual fund is represented by $m(t) = \sin\left(\frac{e}{3}\right)^t$ thousand dollars per year where t is measured in years. Is the amount of money from this mutual fund increasing or decreasing at time $t = 4$ years? Justify your answer.

3. A particle is traveling along the y-axis and its position from the origin can be modeled by
$$y(t) = 6t - 2t^3 + 10$$
where y is meters and t is minutes.
   a. On the interval $0 \le t \le 2$, when is the particle farthest above the origin.

   b. On the interval $0 \le t \le 2$, what is the particle's maximum speed?

4. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 36 - x^2$. What length and width should the rectangle have so that its area is a maximum?

5. The graph shows the derivative of f, f'. Identify the intervals when f is increasing and decreasing. Include a justification statement.



Increasing: Decreasing:

6. For the table below, selected values of x and $f(x)$ are given. Assume that $f'(x)$ and $f''(x)$ do not change signs.

x	$f(x)$
0	-10
1	-8
2	-5
3	-1

a. Is $f(x)$ increasing or decreasing?

b. Is $f(x)$ concave up or concave down?

7. Given the function $g(x) = -x^4 + 2x^2 - 1$, find the interval(s) when g is **concave up** and **decreasing** at the same time.

8. The Mean Value Theorem can be applied to which of the following function on the closed interval $[0, 5]$?

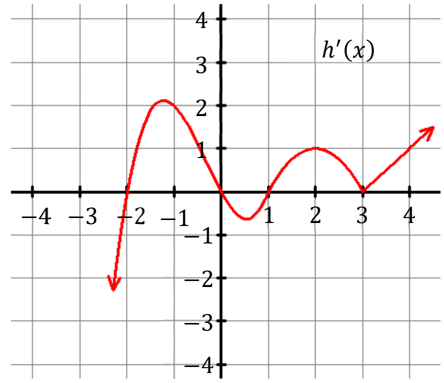
(A) $f(x) = \frac{x-3}{x+3}$

(B) $f(x) = (x-1)^{\frac{2}{3}}$

(C) $f(x) = \frac{x+3}{x-3}$

(D) $f(x) = |x - 4|$

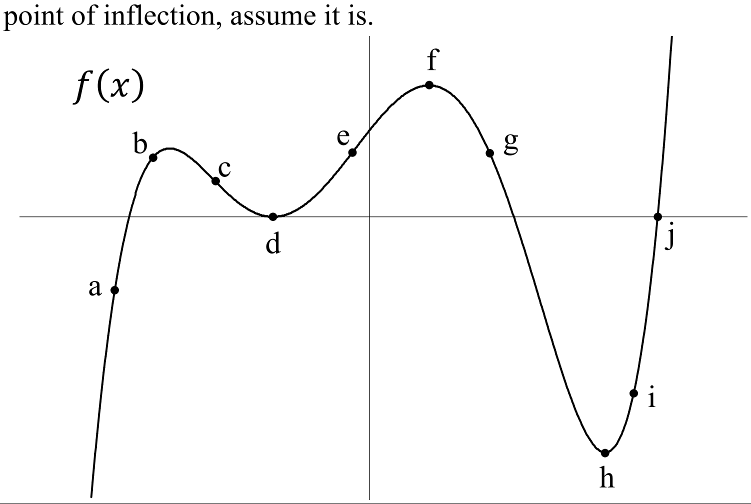
9. To the right is the graph of $h'(x)$. Identify all extrema of $h(x)$. No justification necessary on this problem.



10. The derivative of g is given by $g'(x) = (5 - x)x^{-3}$ for $x > 0$. Find all relative extrema and justify your conclusions.

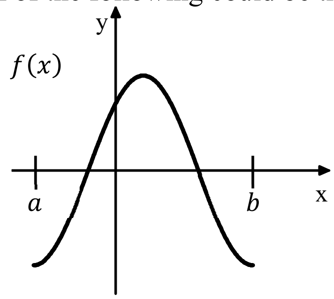
11. Consider the function f defined by $f(x) = e^x \sin x$ with domain $[0, 2\pi]$. Find the absolute maximum and minimum values of $f(x)$.

12. Using the figure below, complete the chart by indicating whether each value is positive (+), negative (-), or zero (0) at the indicated points. For these problems, if the point appears to be a max or min, assume it is. If it appears to be a point of inflection, assume it is.

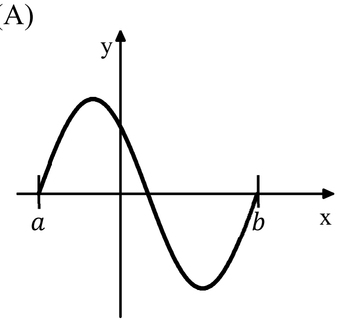


x	a	b	c	d	e	f	g	h	i	j
$f(x)$										
$f'(x)$										
$f''(x)$										

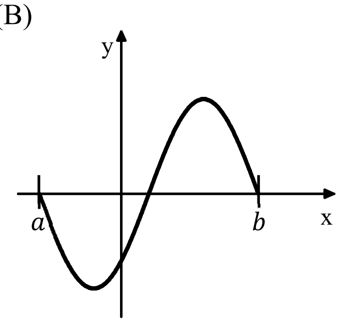
13. The graph of f is shown below. Which of the following could be the graph of the derivative of f?



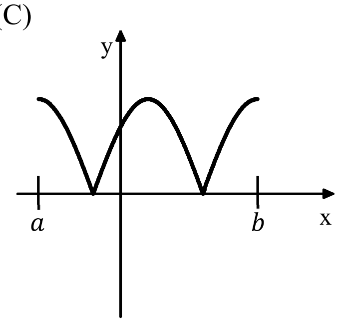
(A)



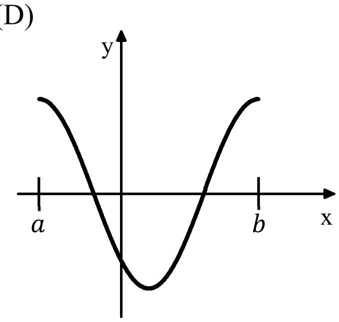
(B)



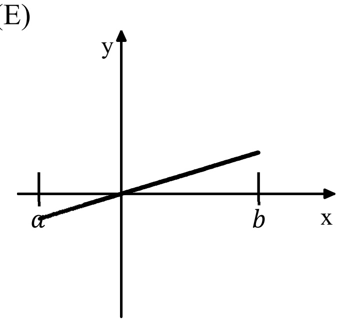
(C)



(D)



(E)



Answers

1. 4	2. Increasing because the rate $m(4)$ is positive. $m(4) \approx 0.3838$	3a. $y(0) = 10$ $\boldsymbol{y(1) = 14}$ $y(2) = 6$ At $t = 1$ minutes	3b. $y'(0) = 6$ $y'(2) = -18$ 18 meters / minute	4. $2\sqrt{12} \times 24$

5. Increasing on the interval $(-3, -1)$ and $(3, \infty)$. Decreasing on the interval $(-\infty, -3)$ and $(-1, 3)$.	6a. Increasing	6b. Concave up	7. $\left(-\sqrt{\frac{1}{3}}, 0\right)$	8. A

9. Min at $x = -2$ and $x = 1$. Max at $x = 0$.	10. Relative maximum at $x = 5$ because g' changes sign from positive to negative.	11. $g(0) = 0$ $\boldsymbol{g\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)}$ **ABS MAX** $\boldsymbol{g\left(\frac{7\pi}{4}\right) = -e^{\frac{7\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)}$ **ABS MIN** $g(2\pi) = 0$

12.

$\boldsymbol{x}$	$\boldsymbol{a}$	$\boldsymbol{b}$	$\boldsymbol{c}$	$\boldsymbol{d}$	$\boldsymbol{e}$	$\boldsymbol{f}$	$\boldsymbol{g}$	$\boldsymbol{h}$	$\boldsymbol{i}$	$\boldsymbol{j}$
$\boldsymbol{f(x)}$	$-$	$+$	$+$	0	$+$	$+$	$+$	$-$	$-$	0
$\boldsymbol{f'(x)}$	$+$	$+$	$-$	0	$+$	0	$-$	0	$+$	$+$
$\boldsymbol{f''(x)}$	$-$	$-$	0	$+$	0	$-$	$-$	$+$	$+$	$+$

13. A