5.7 The Second Derivative Test

Calculus

Practice

Find the relative extrema by using the Second Derivative Test. Justify your answer.

1. $f(x) = 5 + 3x^2 - x^3$

CV: $x = 0, 2$

$f'(x) = 6x - 3x^2$
$f'(x) = 3x(2 - x)$
$f' = 0$ @ $x = 0, x = 2$

$f''(x) = 6 - 6x$
$f''(x) = 6(1 - x)$

2nd Derivative Test

$f''(0) = 6(1-0) > 0$ concave up
$f''(2) = 6(1-2) < 0$ concave down

- $f'(0) = 0$ & $f''(0) > 0$
  ∴ f has relative minimum @ $x = 0$
- $f'(2) = 0$ & $f''(2) < 0$
  ∴ f has relative maximum @ $x = 2$

2. $h(x) = (2x - 5)^2$

CV: $x = 5/2$

$h'(x) = 2(2x-5)(2)$
$h' = 0$ @ $x = 5/2$

$h''(x) = 2(2)(2)$

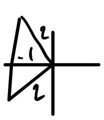
2nd Derivative Test

$h''(5/2) = 8 > 0$ concave up

- $h'(5/2) = 0$ and $h''(5/2) > 0$
  ∴ h has a relative minimum @ $x = 5/2$

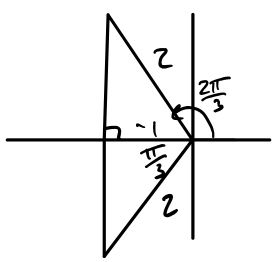
3. $g(x) = x + 2\sin x$ on the interval $(0, 2\pi)$

CV: $x = \pi/3, 4\pi/3$



$g'(x) = 1 + 2\cos x$

$g'(x) = 0$
$0 = 1 + 2\cos x$
$-\frac{1}{2} = \cos x$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$



$g''(x) = -2\sin x$

2nd Derivative Test

$g''\left(\frac{2\pi}{3}\right) = -2\sin\left(\frac{2\pi}{3}\right) < 0$ concave down
$g''\left(\frac{4\pi}{3}\right) = -2\sin\left(\frac{4\pi}{3}\right) > 0$ concave up

- $g'\left(\frac{2\pi}{3}\right) = 0$ & $g''\left(\frac{2\pi}{3}\right) < 0$
  ∴ g has a local maximum @ $x = \frac{2\pi}{3}$
- $g'\left(\frac{4\pi}{3}\right) = 0$ & $g''\left(\frac{4\pi}{3}\right) > 0$
  ∴ g has a local minimum @ $x = \frac{4\pi}{3}$

4. $f(x) = 2x^4 - 8x + 3$

CV: $x = 1$

$f' = 8x^3 - 8$
$f' = 0$
$8x^3 - 8 = 0$
$x^3 = 1$
$x = 1$

$f'' = 24x^2$

2nd Derivative Test

$f''(1) = 24(1)^2 > 0$ concave up

- $f' = 0$ & $f''(1) > 0$
  ∴ f has a relative minimum at $x = 1$

5.7 The Second Derivative Test

Test Prep

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

CV: $x=0, x=3/2$

$f' = 4x^3 - 6x^2$
$f' = 0$
$0 = 2x^2(2x-3)$
$x=0,\ x=3/2$

$f' = 2x^2(2x-3)$

f': $x=-10$ | $x=1$ | $x=10$
NEG 0 NEG 3/2 POS
R.MIN

CV: $x=0, x=1$

$f'' = 12x^2 - 12x$
$f''=0$
$0 = 12x(x-1)$
$x=0,\ X=1$

$f'' = 12x(x-1)$

f'': $x=-1$ | $x=1/2$ | $x=2$
POS 0 NEG 1 POS
IP IP

(A) The graph of the function has two points of inflection, and the function has one relative extremum.

(B) The graph of the function has one point of inflection, and the function has two relative extrema.

(C) The graph of the function has two points of inflection, and the function has two relative extrema.

(D) The graph of the function has two points of inflection, and the function has three relative extrema.

(E) The function has no relative extremum.

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6. At what value(s) of x does $f(x) = x^4 - 8x^2$ have a relative minimum?

CV: $x=-2,0,2$

$f' = 4x^3 - 16x$
$f'=0$
$0 = 4x(x^2-4)$
$x=0,\ x=-2,\ x=2$

$f'' = 12x^2 - 16$

$f''(-2) = 12(-2)^2 - 16 > 0$ concave up
$f''(0) = 12(0)^2 - 16 < 0$ concave down
$f''(2) = 12(2)^2 - 16 > 0$ concave up

2nd Derivative Test

(A) 0 and -2 only (B) 0 and 2 only (C) 0 only

(D) -2 and 2 only (E) $-2, 0,$ and 2 only

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7. What is the maximum value of the *derivative* of $f(x) = 3x^2 - x^3$?

Y-value

Parabola
ABS MAX = Rel MAX

$f' = 6x - 3x^2$ FIND MAX of f'

CV: $x=1$

$f'' = 6 - 6x$
$f''=0$
$0 = 6(1-x)$
$x=1$

$f'(1) = 6(1) - 3(1)^2$
$f'(1) = 3$ MAX value

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4