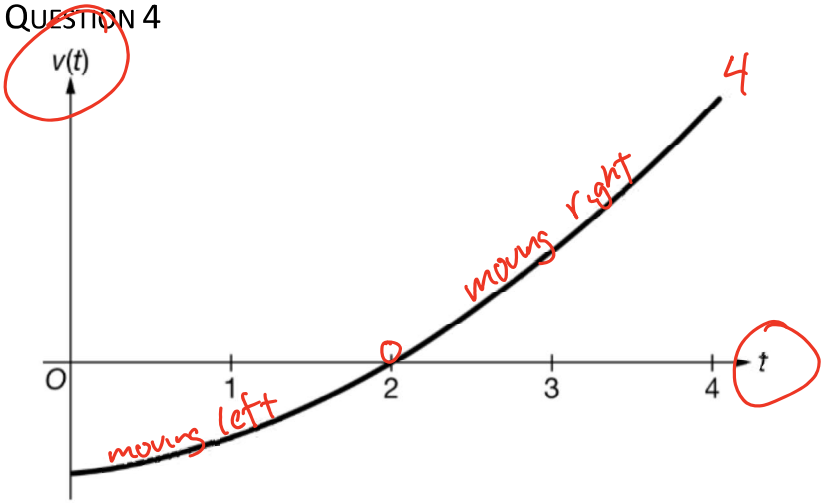
Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

TOPIC QUESTION 4



A particle traveling on the x-axis has position $x(t)$ at time t. The graph of the particle's velocity $v(t)$ is shown above for $0 \le t \le 4$. Which of the following expressions gives the total distance traveled by the particle over the time interval $0 \le t \le 4$?

Ⓐ $x(0) - x(4)$

Ⓑ $x(4) - x(0)$

Ⓒ $(x(0) - x(2)) + (x(4) - x(2))$

Ⓓ $(x(2) - x(0)) + (x(2) - x(4))$

Total Distance = $|x(2) - x(0)| + |x(4) - x(2)|$

negative

positive

This question gets easier when we learn to integrate (much easier)

TOPIC QUESTION 5

An object moves along a straight line so that at any time t, $0 \le t \le 9$, its position is given by $x(t) = 7 + 6t - t^2$. For what value of t is the object at rest?

Ⓐ $t = 3$

Ⓑ $t = 6$

Ⓒ $t = \frac{13}{2}$

Ⓓ $t = 7$

$v(t) = 0$ the object is at rest.

$v = x' = 6 - 2t$

$0 = 6 - 2t$

$2t = 6$

$t = 3$

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

TOPIC QUESTION 7

An object moves along a straight line so that at any time t, for $0 \le t \le 8$, its position is given by $x(t) = 5 + 4t - t^2$. For what value of t is the object at rest?

$v(t)=0$

(A) $t = 2$

(B) $t = 4$

(C) $t = \frac{9}{2}$

(D) $t = 5$

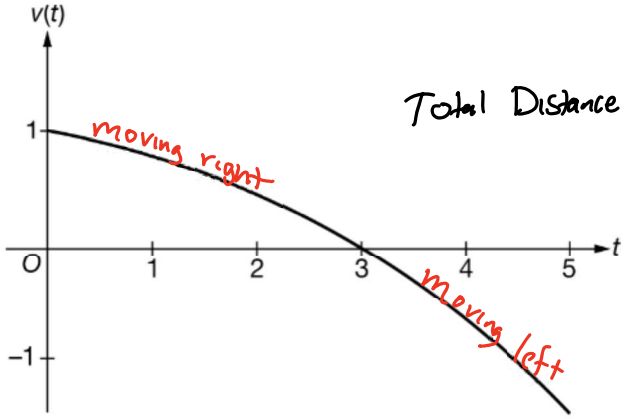
$v(t) = 4 - 2t$

$0 = 4 - 2t$

$2t = 4$

$t = 2$

TOPIC QUESTION 8



Total Distance = $|x(3) - x(0)| + |x(5) - x(3)|$

$x(5) - x(3)$: negative

$|x(5) - x(3)|$: positive

A particle traveling on the x-axis has position $x(t)$ at time t. The graph of the particle's velocity $v(t)$ is shown above for $0 \le t \le 5$. Which of the following expressions gives the total distance traveled by the particle over the time interval $0 \le t \le 5$?

(A) $x(0) - x(5)$

(B) $x(5) - x(0)$

(C) $(x(3) - x(0)) + (x(3) - x(5))$

(D) $(x(0) - x(3)) + (x(5) - x(3))$

This question gets easier when we learn to integrate (much easier)

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM MC 7

If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45

(B) -30

(C) -15

(D) -10

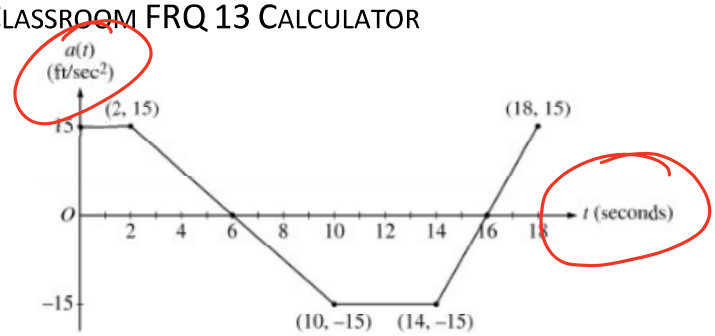
(E) -5

$$AV = \frac{[-5(3)^2]-[-5(0)^2]}{3-0}$$

$$= \frac{-45}{3}$$

$$AV = -15$$

APCLASSROOM FRQ 13 CALCULATOR



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above

Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

$v'(2) = a(2) = 15$

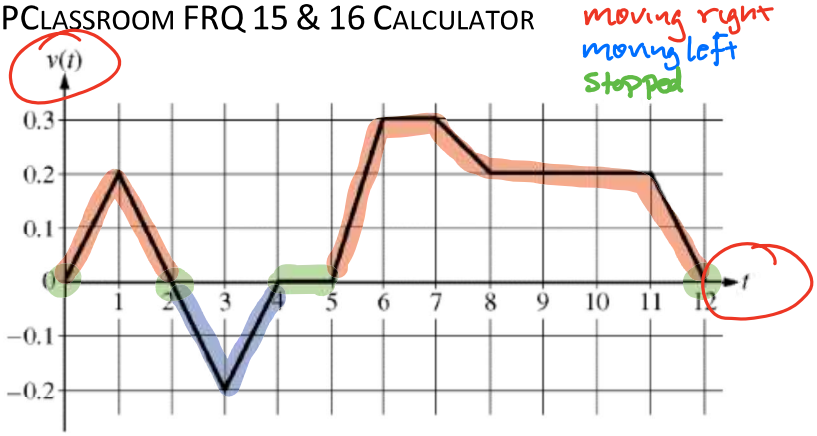
$\therefore a(t) > 0$ at $t = 2$

$\therefore$ velocity is increasing at $t = 2$

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM FRQ 15 & 16 CALCULATOR

moving right
moving left
stopped



Caren rides her bicycle along a straight road from home to school, starting at home at time t=0 minutes and arriving at school at time t=12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

a. Find the acceleration of Caren's bicycle at time t=7.5 minutes. Indicate units of measure.

(Velocity is linear at $t=7.5$) $a(7.5) = v'(7.5) = \frac{v(7)-v(8)}{7-8} = \frac{0.3-0.2}{-1} = \frac{.1}{-1} = -0.1 \text{ mi/min}^2$

b. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

Caren's velocity changes from positive to negative at $t=2$ minutes.

∴ Caren turns around to go back home at this time.

APCLASSROOM MC 17 CALCULATOR

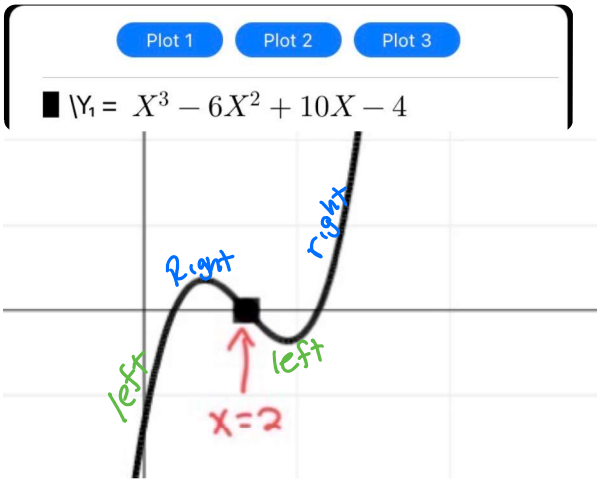
For $t \ge 0$, the velocity of a particle moving along the x-axis is given by $v(t) = t^3 - 6t^2 + 10t - 4$. At what time t does the direction of motion of the particle change from right to left?

(A) 0.586

(B) 1.184

(C) 2.000

(D) 2.816



Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM MC 18 CALCULATOR

Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

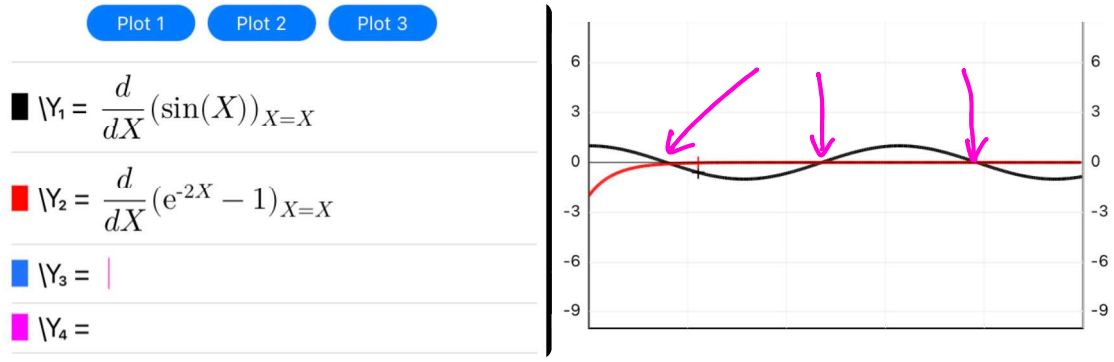
(A) None

(B) One

(C) Two

(D) Three

(E) Four



APCLASSROOM MC 19 CALCULATOR

A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$

(A) -2.016

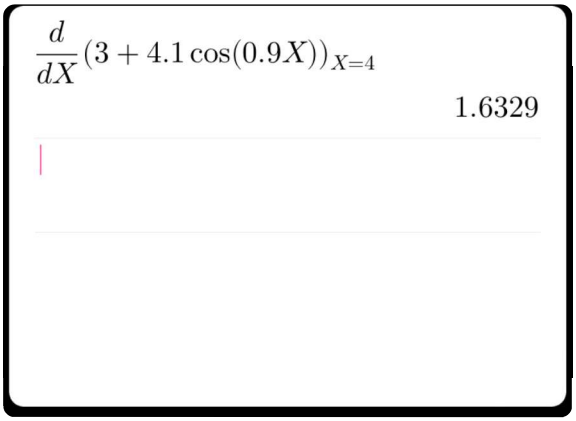
(B) -0.677

(C) 1.633

(D) 1.814

(E) 2.978

$a(4) = v'(4)$



APCLASSROOM MC 20 CALCULATOR

A particle moves along a straight line with velocity given by $v(t) = 5 + e^{t/3}$ for time $t \ge 0$. What is the acceleration of the particle at time $t = 4$?

(A) 0.422

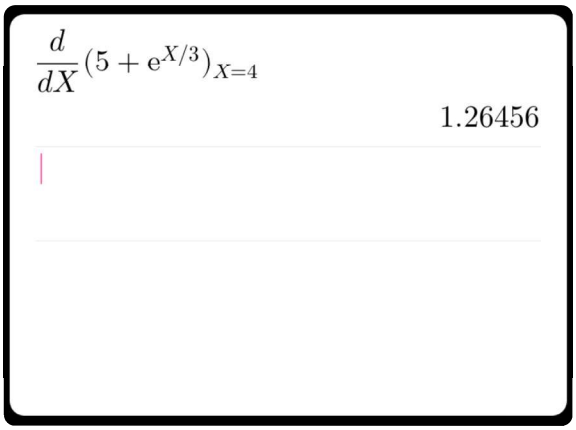
(B) 0.698

(C) 1.265

(D) 8.794

(E) 28.381

$a(4) = v'(4)$



Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM MC 21 CALCULATOR

A particle moves along the *x*-axis so that at time $t \geq 0$ the position of the particle is given by $x(t) = 0.5t^4 - 1.5t^3 - 2t^2 + 6t - 1$. What is the velocity of the particle at the first instance the particle is at the origin?

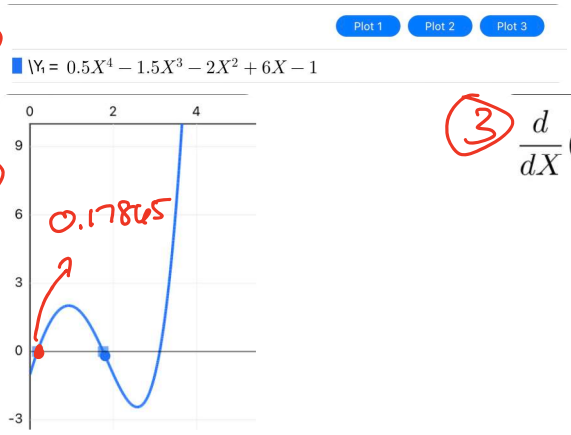
(A) -4.071

(B) -2.048

(C) 0

(D) 5.153

(E) 6



$\frac{d}{dX}(Y_1)_{X=.17865}$

5.15318

APCLASSROOM MC 22 CALCULATOR

A particle moves along the x-axis so that its position at time $t > 0$ is given by $x(t)$ and $\frac{dx}{dt} = -10t^4 + 9t^2 + 8t$. The acceleration of the particle is zero when t =

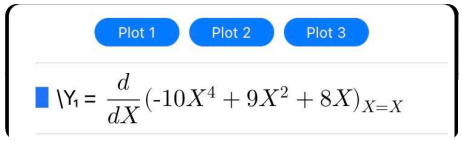
(A) 0.387

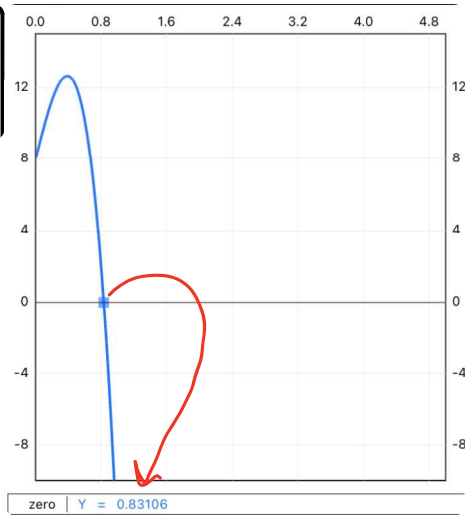
(B) 0.831

(C) 1.243

(D) 1.647

(E) 8.094





APCLASSROOM MC 23

The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0

(B) 2

(C) 4

(D) 8

(E) 12

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM MC 25 CALCULATOR

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity ofthe particle when its acceleration is zero?

$-\frac{1}{2}t^2$

FIND V(b) where a(b) = 0

(A) -5.19

(B) 0.74

(C) 1.32

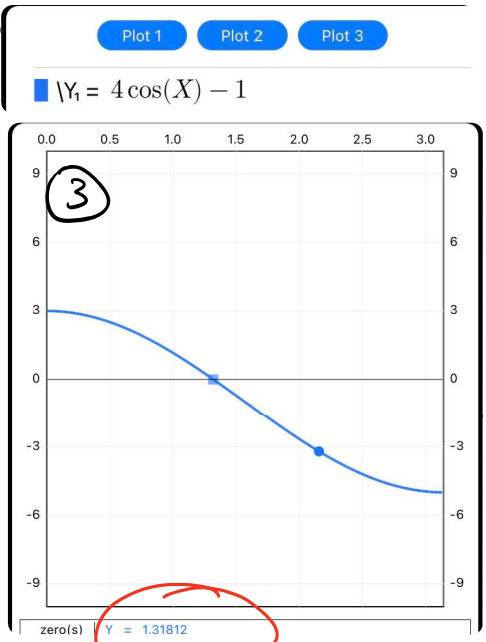
(D) 2.55

(E) 8.13

(1) $v = 4\sin t - t$

$a = 4\cos t - 1$

(2)

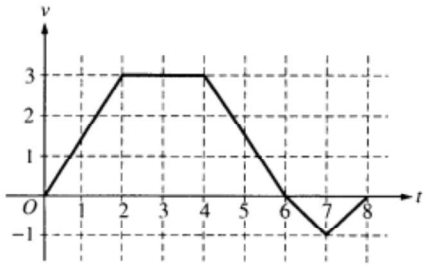


(4)

$4\sin(1.31812) - 1.31812$

2.5548672745

APCLASSROOM MC 26



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

(A) 2

(B) 4

(C) 6 b/c velocity changes from positive to negative at t = 6.

(D) 7

(E) 8

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM MC 27 CALCULATOR

The position of a particle moving along the x-axis is given by a twice-differentiable function x(t). If x(2) < 0, x'(2) < 0, and x"(2) <0, which of the following statements must be true about the particle at time t = 2 ?

$x'(2) < 0$

(A) The particle is moving toward the origin at a decreasing speed.

$x'(2) < 0$

(B) The particle is moving toward the origin at an increasing speed.

(C) The particle is moving away from the origin at a decreasing speed.

is on left and moving left — $v(t)$ and $a(t)$ are same sign

(D) The particle is moving away from the origin at an increasing speed.

(E) The particle is moving away from the origin at a constant speed.

$x(2) < 0$ left of origin

$x'(2) < 0$ moves left

$x''(2) < 0$ increasing speed

moving away from origin

positive

$v < 0$

$x'(2) < 0$

$x(2)$

Collective slopes are decreasing.

APCLASSROOM MC 28 CALCULATOR

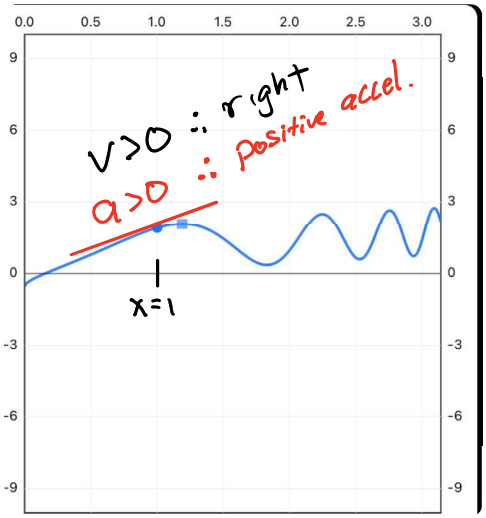
The velocity of a particle moving along the *x*-axis is given by $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. Which of the following statements describes the motion of the particle at $t = 1$?

(A) The particle is moving to the left with positive acceleration.

(B) The particle is moving to the right with positive acceleration.

(C) The particle is moving to the left with negative acceleration.

(D) The particle is moving to the right with negative acceleration.



Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM FRQ 30

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position x=7 meters when t=0 seconds.

$x(0) = 7$

ARC

Estimate the acceleration of the particle at t=36 seconds. Show the computations that lead to your answer. Indicate units of measure.

$$a(36) = v'(36) \approx \frac{v(32) - v(40)}{32 - 40}$$

$$\approx \frac{-4 - 7}{-8}$$

$$\approx \frac{11}{8} \text{ m/sec}^2$$

APCLASSROOM FRQ 31

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position x=7 meters when t=0 seconds.

For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

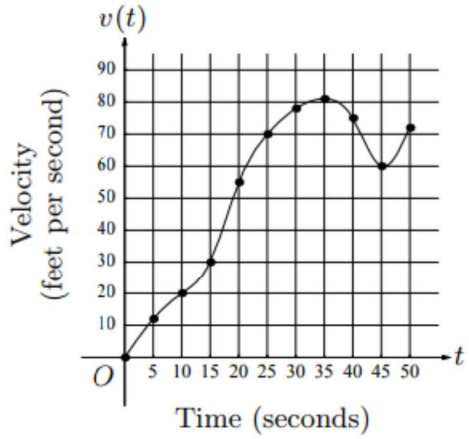
- $v(8) = 5$ and $v(20) = -10$; $v(32) = -4$ and $v(40) = 7$

  $\therefore$ $v(t)$ changes sign on $(8, 20)$ and on $(32, 40)$

  $\therefore$ $v(t)$ changes directions on $(8, 20)$ and on $(32, 40)$

Unit 4.2 Straight-Line Motion: Connecting Position, Velocity & Acceleration

APCLASSROOM FRQ 49 & 50 CALCULATOR



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

a. During what intervals of time is the acceleration of the car positive? Give a reason for your answer

The velocity is increasing on $(0, 35)$ and $(45, 50)$
∴ the acceleration is positive on these intervals.

b. Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

$$\text{Average Acceleration} \approx \frac{v(0) - v(50)}{0 - 50}$$

$$\approx \frac{0 - 72}{-50}$$

$$\approx \frac{72}{50} \text{ ft/sec}^2$$