Unit 4.3 Rates of Change in Applied Context (Nonmotion Problems)

APCLASSROOM FRQ 1

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then $\frac{db}{dt} = \frac{1}{5}(100 - B)$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

B=40 B=70

+1

$$\left.\frac{db}{dt}\right|_{B=40} = \frac{1}{5}(100-40) = \frac{1}{5}(60) = 12$$

$$\left.\frac{db}{dt}\right|_{B=70} = \frac{1}{5}(100-70) = \frac{1}{5}(30) = 6$$

$$\left.\frac{db}{dt}\right|_{B=40} > \left.\frac{db}{dt}\right|_{B=70}$$

+1

$\therefore$ the bird is gaining weight faster when it weighs 40 grams

APCLASSROOM MC 2 CALCULATOR

People are entering a building at a rate modeled by $f(t)$ people per hour and exiting the building at a rate modeled by $g(t)$ people per hour, where t is measured in hours. The functions f and g are nonnegative and differentiable for all times t. Which of the following inequalities indicates that the rate of change of the number of people in the building is increasing at time t?

f(t) enter
g(t) exit

Total people = f(t) - g(t)

$$\frac{dp}{dt} = f' - g'$$

Ⓐ $f(t) > 0$

Ⓑ $f'(t) > 0$

Ⓒ $f(t) - g(t) > 0$

Ⓓ $f'(t) - g'(t) > 0$

APCLASSROOM MC 3

If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

Ⓐ $\frac{dP}{dt} = 200$ Constant slope implies P is linear

Ⓑ $\frac{dP}{dt} = 200t$ Linear slope implies P is QUAD

Ⓒ $\frac{dP}{dt} = 100t^2$ QUAD slope implies P is Cubic

Ⓓ $\frac{dP}{dt} = 200P$

Ⓔ $\frac{dP}{dt} = 200P^2$

Unit 4.3 Rates of Change in Applied Context (Nonmotion Problems)

APCLASSROOM MC 7 CALCULATOR

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

37 58

The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t, in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

$v(t) > 0$

$a(t) > 0$

$37 < v(3) < 58$

(A) 37

(B) 49

(C) 58

(D) 65

(E) 92

APCLASSROOM FRQ 9 CALCULATOR

km/hr

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x(1 - e^{-x/2})$. liters/km

Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

Reference only

Plot 1 Plot 2 Plot 3

$Y_1 = 0.05X(1 - e^{-X/2})$

$Y_2 = 120(1 - e^{-10X^2})$

$\underbrace{\frac{d}{dX}(Y_1)_{X=2}}_{g'(2)}\underbrace{Y_2(2)}_{r(2)}$

6

$\frac{dg}{dt} = \underbrace{g'(x)}_{\text{LINK}} \cdot \underbrace{\frac{dx}{dt}}_{\text{LINK}} = g' \cdot r(t)$ +2

$\left.\frac{dg}{dt}\right|_{t=2} = g'(2) \cdot r(2) = 6$ liters/hour +1

APCLASSROOM FRQ 12 CALCULATOR

The following are related to this scenario:

A store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function S defined by $S(t)=0.5t^4-16t^3+144t^2$ for $0 \leq t \leq 12$. At time $t=0$, when the sale begins, there are no shoppers in the store.

At what rate are shoppers entering the store 3 hours after the start of the sale?

$S'(3) = 486$ shoppers/hour +1

$\frac{d}{dX}(0.5X^4 - 16X^3 + 144X^2)_{X=3}$

486.00024

Reference Only

Unit 4.3 Rates of Change in Applied Context (Nonmotion Problems)

APCLASSROOM FRQ 22 CALCULATOR

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$B(t) = 1000e^{0.06t}$ penguins per year

and the death rate for the penguins on the island is modeled by

$D(t) = 250e^{0.1t}$ penguins per year.

What is the rate of change for the penguin population on the island at time $t = 0$?

Reference only

$$\boxed{e^0 = 1}$$

$$P'(t) = B(t) - D(t)$$

$$P'(0) = B(0) - D(0)$$

$$= 1000e^{0.06(0)} - 250e^{0.1(0)}$$

$$= 1000e^0 - 250e^0$$

$$P'(0) = 750 \text{ penguins/year}$$ +1

APCLASSROOM FRQ 23 PART A AND PART C CALCULATOR

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(5) = -24.587$ t/h² (hour) +1

Reference only

$$\frac{d}{dX}(90 + 45\cos(X^2/18))_{X=5}$$

-24.58744

+1 At 5 hours, the rate at which unprocessed gravel arrives at a gravel processing plant is decreasing by 24.587 tons per hour each hour.

c. Is the amount of unprocessed gravel at the plant increasing or decreasing at time t = 5 hours? Show the work that leads to your answer.

Adding unprocessed gravel	Processing gravel
$G(5) = 98.141$ tons/hour	100 tons/hour

$G(5) < 100$ +1

∴ The amount of unprocessed gravel is decreasing at $t = 5$ hours. +1

Reference only

$$90 + 45\cos(5^2/18)$$

98.1407640311