

### Unit 4.4 Introduction to Related Rates

Differentiate with respect to t.

<p>1. <math>P = 4x</math>  <math>\frac{dP}{dt} = 4 \frac{dx}{dt}</math></p>	<p>2. <math>A = bh</math>  <math>\frac{dA}{dt} = \frac{db}{dt}h + b \frac{dh}{dt}</math>  <i>Product Rule</i></p>	<p>3. <math>A = \frac{1}{2}bh</math>  <math>\frac{dA}{dt} = \frac{1}{2} \frac{db}{dt}h + \frac{1}{2}b \frac{dh}{dt}</math>  <i>Product Rule</i></p>
<p>4. <math>C = 2\pi r</math>  <math>\frac{dC}{dt} = 2\pi \frac{dr}{dt}</math></p>	<p>5. <math>A = \pi r^2</math>  <math>\frac{dA}{dt} = 2\pi r \frac{dr}{dt}</math></p>	<p>6. <math>V = x^3</math>  <math>\frac{dV}{dt} = 3x^2 \frac{dx}{dt}</math></p>
<p>7. <math>x^2 + y^2 = z^2</math>  <math>2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}</math></p>		<p>8. <math>V = x^2h</math>  <math>\frac{dV}{dt} = 2x \frac{dx}{dt}h + x^2 \frac{dh}{dt}</math>  <i>Product Rule</i></p>
<p>9. <math>V = \frac{1}{3}\pi r^2h</math>  <math>\frac{dV}{dt} = \frac{2}{3}\pi r \cdot \frac{dr}{dt}h + \frac{1}{3}\pi r^2 \frac{dh}{dt}</math>  <i>Product Rule</i></p>	<p>10. <math>f(x) = -5x^2 + 30x + 50</math>  <math>\frac{df}{dx} = -10x \frac{dx}{dt} + 30 \frac{dx}{dt}</math></p>	
<p>11. <math>V = \frac{4}{3}\pi r^3</math>  <math>\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}</math></p>	<p>12. <math>f(p) = t</math>  <math>f'(p) \cdot \frac{dp}{dt} = 1</math></p>	<p>13. <math>x(t) = -5t^2 + 30t + 50</math>  <math>x'(t) = -10t + 30</math></p>
<p>14. <math>V = lwh</math>  <math>\frac{dV}{dt} = \frac{dl}{dt}wh + l \cdot \left[ \frac{dw}{dt}h + w \frac{dh}{dt} \right]</math>  <math>= \frac{dl}{dt}wh + l h \frac{dw}{dt} + l w \frac{dh}{dt}</math></p>		
<p>15. <math>\cos \theta = \frac{a}{h}</math>  <math>-\sin \theta \cdot \frac{d\theta}{dt} = \frac{\frac{da}{dt}h - a \frac{dh}{dt}}{h^2}</math></p>		
<p>16. <math>\frac{dW}{dt} = \frac{1}{4}(100 - W)</math> Find the second derivative, <math>\frac{d^2W}{dt^2}</math>, and write it in terms of W.  <math>= 25 - \frac{1}{4}W</math>  <math>\frac{d^2W}{dt^2} = -\frac{1}{4} \frac{dW}{dt} = -\frac{1}{4} \left( 25 - \frac{1}{4}W \right)</math></p>		

17. Differentiate  $V = \pi r^2h$  with respect to t if  $h$  is constant.  $h' = 1$

$$\frac{dV}{dt} = 2\pi r h \cdot \frac{dr}{dt}$$

18. Differentiate  $V = \pi r^2h$  with respect to t if  $r$  is constant.  $r' = 1$

$$\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$$