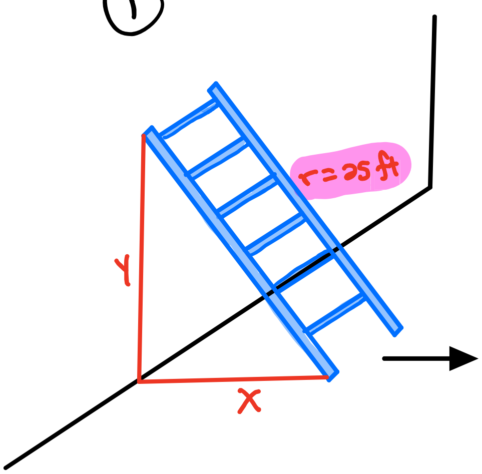
Unit 4.5.1 Right Triangles Solving Related Rates Problems

Right Triangles – Falling Ladder

1. A ladder is 25 feet long and is leaning against the wall of a house. When the base has slid 15 feet from the house, it is moving horizontally at a rate of 6 ft/sec. How fast is the top of the ladder moving down the wall when its base is 15 feet from the wall?

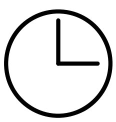
①



$$x^2 + y^2 = 25^2$$

$$\frac{d}{dt}x^2 + \frac{d}{dt}y^2 = \frac{d}{dt}25^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



FIND $\frac{dy}{dt}$

$\frac{dx}{dt} = 6$ ft/sec

$x = 15$

Always $r = 25$

$$15^2 + y^2 = 25^2$$
$$225 + y^2 = 625$$
$$y^2 = 400$$
$$y = 20$$

$$2(15)(6) + 2(20)\frac{dy}{dt} = 0$$

$$180 + 40\frac{dy}{dt} = 0$$

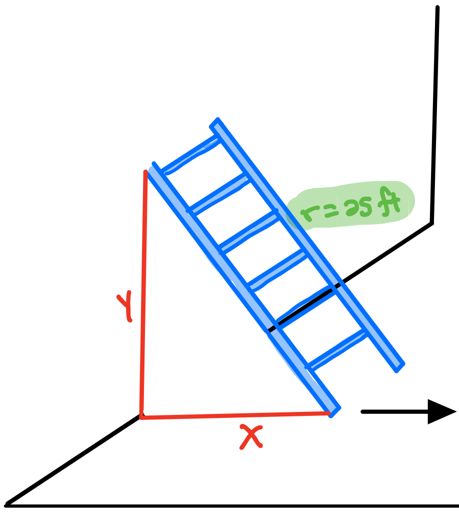
$$40\frac{dy}{dt} = -180$$

$$\frac{dy}{dt} = \frac{-180}{40}$$

$$\frac{dy}{dt} = -4.5 \text{ ft/sec}$$

Right Triangles – Falling Ladder

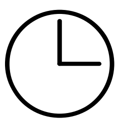
2. A ladder is 25 feet long and is leaning against the wall of a house. When the base has slid 9 feet from the house, it is moving horizontally at a rate of 4 ft/sec. Find the rate at which the area of the triangle is changing when the ladder is 9 feet from the wall.



$r = 25$

$$A = \frac{1}{2}xy$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}y + \frac{1}{2}x\cdot\frac{dy}{dt}$$



FIND $\frac{dA}{dt}$

when $x = 9$

Always $r = 25$

$\frac{dx}{dt} = 4$ ft/sec

$$\frac{dA}{dt} = \left[\frac{1}{2}(4)(\sqrt{544}) + \frac{1}{2}(9)\left(\frac{-36}{\sqrt{544}}\right)\right] \text{ ft}^2/\text{min}$$

or

$$\frac{dA}{dt} \approx 39.702 \text{ ft}^2/\text{sec}$$

y, 25, 9

$y^2 + 9^2 = 25^2$

$y^2 = 625 - 81$

$y^2 = \sqrt{544}$

$$x^2 + y^2 = 25^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(9)(4) + 2\sqrt{544}\frac{dy}{dt} = 0$$

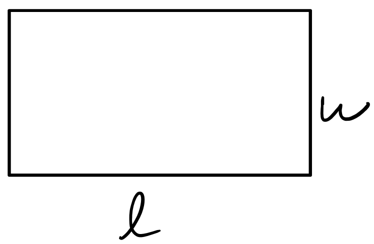
$$\frac{dy}{dt} = \frac{-72}{2\sqrt{544}}$$

$$\frac{dy}{dt} = \frac{-36}{\sqrt{544}}$$

Unit 4.5.1 Rectangles Solving Related Rates Problems

Rectangle – Area

3. The area of a rectangle is increasing at a rate of 15 feet^2/ minute. If the width is increasing at a rate of 2 feet / minute when the length is 4 feet and the width is 3 feet, find the rate of change of the length.

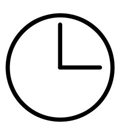


$\frac{dA}{dt} = 15\ ft^2/min$

$A = lw$

$\frac{dA}{dt} = \frac{dl}{dt} w + l \cdot \frac{dw}{dt}$

$15 = \frac{dl}{dt} w + l \cdot \frac{dw}{dt}$



$\frac{dw}{dt} = 2\ ft/min$

$l = 4\ ft$

$w = 3\ ft$

FIND $\frac{dl}{dt}$

$15 = \frac{dl}{dt}(3) + 4(2)$

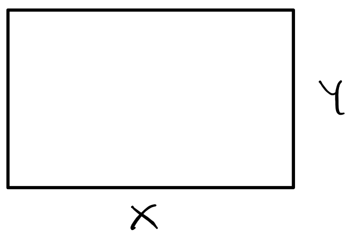
$15 = \frac{dl}{dt} \cdot 3 + 8$

$7 = \frac{dl}{dt} \cdot 3$

$\frac{7}{3}\ ft/min = \frac{dl}{dt}$

Rectangle – Perimeter & Area

4. The length x of a rectangle is decreasing at a rate of 4 cm/minute and the width y is increasing at the rate of 3 cm/minute. When $x = 12$ cm and $y = 8$ cm, find the rate of change of (a) the perimeter and (b) the area of the rectangle.



$\frac{dx}{dt} = -4$ cm/min

$\frac{dy}{dt} = 3$ cm/min

$P = 2x + 2y$

$\frac{dP}{dt} = 2 \cdot \frac{dx}{dt} + 2\frac{dy}{dt}$

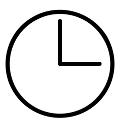
$\frac{dP}{dt} = 2(-4) + 2(3)$

$\frac{dP}{dt} = -2$ cm/min

$A = xy$

$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$

$\frac{dA}{dt} = (-4)y + x(3)$



$x = 12$

$y = 8$

a) FIND $\frac{dP}{dt}$

b) FIND $\frac{dA}{dt}$

$\frac{dA}{dt} = -4(8) + 12(3)$

$\frac{dA}{dt} = -32 + 36$

$\frac{dA}{dt} = 4$ cm²/min