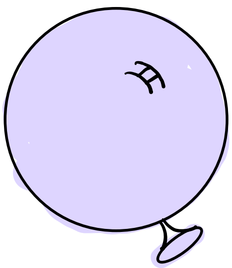
Unit 4.5.2 Spheres Solving Related Rates Problems

Spheres – Balloons Galore

1. A spherical balloon is inflated with helium at the rate of 100π ft^3/min. (Volume of a sphere is $v = \frac{4}{3}\pi r^3$, where *r* is the radius. Surface area of a sphere is $A = 4\pi r^2$, where *r* is the radius.)

   a. How fast is the balloon's radius increasing when the radius is 5 feet?



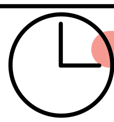
$\frac{dV}{dt} = 100\pi$ ft^3/min

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt} V = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$100\pi = 4\pi r^2 \cdot \frac{dr}{dt}$



$r = 5$

Find $\left.\frac{dr}{dt}\right|_{r=5}$

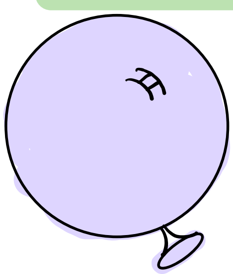
$100\pi = 4\pi(5)^2 \frac{dr}{dt}$

$100\pi = 4\pi(25)\frac{dr}{dt}$

$\frac{100\pi}{100\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = 1$ ft/min

   b. How fast is the surface area increasing when the radius is 5 feet?

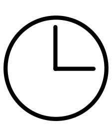


$\frac{dV}{dt} = 100\pi$ ft^3/min

$A = 4\pi r^2$

$\frac{d}{dt} A = \frac{d}{dt}(4\pi r^2)$

$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$



Find $\left.\frac{dA}{dt}\right|_{r=5}$

$r = 5$

$\frac{dV}{dt} = 100\pi$ ft^3/min

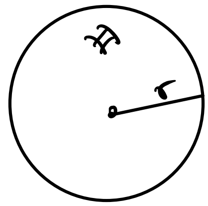
$\left.\frac{dr}{dt}\right|_{r=5} = 1$ ft/min

$\frac{dA}{dt} = 8\pi(5)(1)$

$\frac{dA}{dt} = 40\pi$ ft^2/min

Spheres

2. The radius of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the surface area of the sphere when the radius is 6 inches. (Volume of a sphere is $v = \frac{4}{3}\pi r^3$, where *r* is the radius. Surface area of a sphere is $A = 4\pi r^2$, where *r* is the radius.)



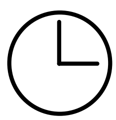
$\frac{dr}{dt} = 2 \text{ in/min}$

$A = 4\pi r^2$

$\frac{d}{dt} A = \frac{d}{dt} 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 8\pi r \cdot (2)$



$r = 6$

FIND $\frac{dA}{dt}$

$\frac{dA}{dt} = 8\pi (6)(2)$

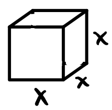
$\frac{dA}{dt} = 96\pi \text{ in}^2/\text{min}$

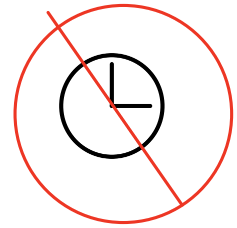
Unit 4.5.2 CUBES Solving Related Rates Problems

Cubes - Volume

3. If the volume of a cube is increasing at a rate of 24 in^3/min and the surface area is increasing at 12 in^2/min, what is the length of each edge of the cube?

x = length of Edge





$\frac{dV}{dt} = 24 \text{ in}^3/\text{min}$

$\frac{dA}{dt} = 12 \text{ in}/\text{min}$

FIND X

EQ

$V = x^3$

$\frac{d}{dt} V = \frac{d}{dt} x^3$

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$24 = 3x^2 \frac{dx}{dt}$

$\frac{24}{3x^2} = \frac{dx}{dt}$

EQ

$A = 6x^2$

$\frac{d}{dt} A = \frac{d}{dt}(6x^2)$

$\frac{dA}{dt} = 12x \cdot \frac{dx}{dt}$

$12 = 12x \frac{dx}{dt}$

$\frac{1}{x} = \frac{dx}{dt}$

System of Equations

1

$\frac{24}{3x^2} = \frac{1}{x}$

$24x = 3x^2$

$0 = 3x^2 - 24x$

$0 = 3x(x-8)$

$0 = 3x$, $\cancel{0 = x}$; $0 = x - 8$, $8 = x$

The length of an edge of the Cube is 8 inches

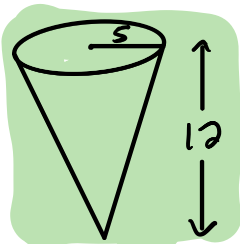
Unit 4.5.2 CONES Solving Related Rates Problems

Cones – Water into a tank

4. A conical tank with the vertex down is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep. (Volume of a cone is $v = \frac{1}{3}\pi r^2 h$, where r is the radius.)

$$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$$

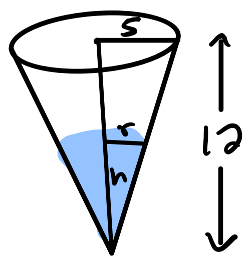
(CAN we substitute into r with h? Yes)



$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{5}{12}$$

$$r = \frac{5}{12}h$$



$$V = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h$$

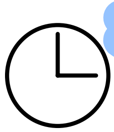
$$V = \frac{1}{3}\pi\left(\frac{25}{144}h^2\right)h$$

$$V = \frac{25\pi}{3 \cdot 144}h^3$$

$$\frac{d}{dt}V = \frac{d}{dt}\left(\frac{25\pi}{3 \cdot 144}h^3\right)$$

$$\frac{dV}{dt} = 3 \cdot \frac{25\pi}{3 \cdot 144}h^2 \cdot \frac{dh}{dt}$$

$$10 = \frac{25\pi}{144}h^2 \cdot \frac{dh}{dt}$$



$h = 8$

FIND $\left.\frac{dh}{dt}\right|_{h=8}$

$$10 = \frac{25\pi}{144}(8)^2 \cdot \frac{dh}{dt}$$

$$\frac{\cancel{10}^{\cancel{2}\,1} \cdot \cancel{144}^{\cancel{18}\,9}}{\cancel{25}_{5} \cdot \pi \cdot \cancel{8}_{1} \cdot \cancel{8}_{\cancel{4}\,2}} = \frac{dh}{dt}$$

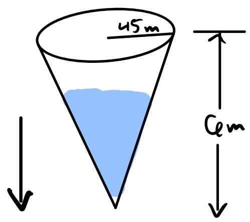
$$\frac{dh}{dt} = \frac{9}{10\pi} \text{ ft/min}$$

Unit 4.5.2 CONES Solving Related Rates Problems

Cones – Water out of a tank

5. Water is flowing at a rate of 50 cubic meters per minute from a concrete conical reservoir. The radius of the reservoir is 45 m and the height is 6 m. (Volume of a cone is $v = \frac{1}{3}\pi r^2 h$, where r is the radius.)

a. How fast is the water level falling when the water is 5 meters deep?



$\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$

$\frac{r}{h} = \frac{45}{6}$

$r = \frac{15}{2}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$V = \frac{1}{\cancel{3}}\pi \cdot \frac{\cancel{15}\cdot 15}{4} h^2 \cdot h$

$V = \frac{5\cdot 15}{4}\pi h^3$

$\frac{d}{dt}V = \frac{d}{dt}\left(\frac{5\cdot 15}{4}\pi h^3\right)$

$\frac{dV}{dt} = \frac{3\cdot 5\cdot 15}{4}\pi h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \cdot \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi h^2 \cdot \frac{dh}{dt}$

$h = 5$

FIND $\left.\frac{dh}{dt}\right|_{h=5}$

$-50 = \frac{225}{4}\pi (5)^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi (25) \frac{dh}{dt}$

(Divide by $\frac{225}{4}\pi(25)$)

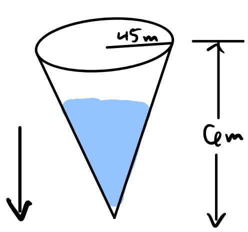
Either Answer

$\frac{dh}{dt} = \frac{-50}{\frac{225}{4}\pi(25)}$ meters/min

$\frac{dh}{dt} = \frac{-8}{225\pi}$ meters/min

b. How fast is the radius of the water's surface changing when the water is 5 meters deep?

Need volume in terms of r...



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \cdot \frac{2}{15} r$

$\frac{d}{dt}\left(V = \frac{1}{3}\cdot\frac{2}{15}\pi r^3\right)$

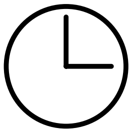
$\frac{r}{h} = \frac{45}{6}$

$r = \frac{15}{2}h$

$h = \frac{2}{15}r$

$\frac{dV}{dt} = \frac{2}{15}\pi r^2 \cdot \frac{dr}{dt}$

$-50 = \frac{2}{15}\pi r^2 \frac{dr}{dt}$

 $h=5$

Find $\left.\dfrac{dr}{dt}\right|_{h=5}$

When $h=5$, $r=\frac{75}{2}$

$$\boxed{\begin{gathered}\frac{2}{15}r=5\\ r=\frac{75}{2}\end{gathered}}$$

$$-50=\frac{2\pi}{15}\left(\frac{75}{2}\right)^2\cdot\frac{dr}{dt}$$

$$-50=\frac{2\pi}{15}\cdot\frac{75\cdot 75}{2\cdot 2}\cdot\frac{dr}{dt}$$

$$\frac{-\cancel{50}\cdot\cancel{15}\cdot\cancel{2}\cdot 2}{\cancel{2}\pi\cdot\cancel{75}\cdot\cancel{75}}=\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{-4}{15\pi}\ \text{meters/min}$$