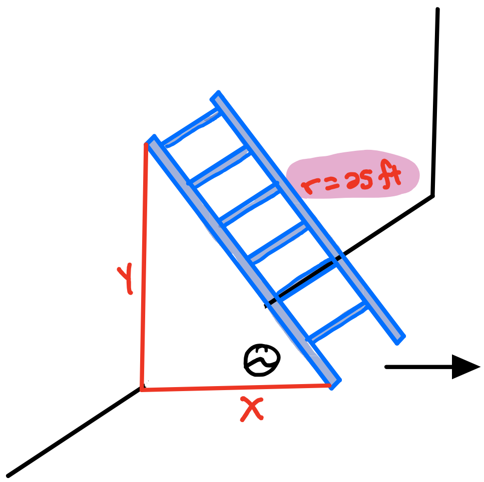
Unit 4.5.3 RIGHT TRIANGLES Solving Related Rates Problems

Right Triangles – Falling Ladder

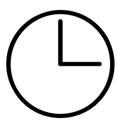
1. A ladder is 25 feet long and is leaning against the wall of a house. When the base has slid 7 feet from the house, it is moving horizontally at a rate of 2 ft/sec. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 7 feet from the wall.

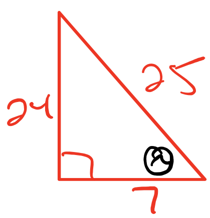


$$\cos\theta = \frac{x}{25}$$

$$\frac{d}{dt}\cos\theta = \frac{d}{dt}\left(\frac{1}{25}x\right)$$

$$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$$





$\boxed{\text{PT-Triple } 7\text{-}24\text{-}25}$

OR

$$\boxed{\begin{aligned} 7^2 + y^2 &= 25^2 \\ 49 + y^2 &= 625 \\ y^2 &= 576 \\ y &= \pm 24 \end{aligned}}$$

$\frac{dx}{dt} = 2 \text{ ft/sec}$

$x = 7$

Find $\frac{d\theta}{dt}$

$y = 24$

$r = 25$

$\sin\theta = \frac{24}{25}$

$$-\left(\frac{24}{25}\right)\frac{d\theta}{dt} = \frac{1}{25}(2)$$

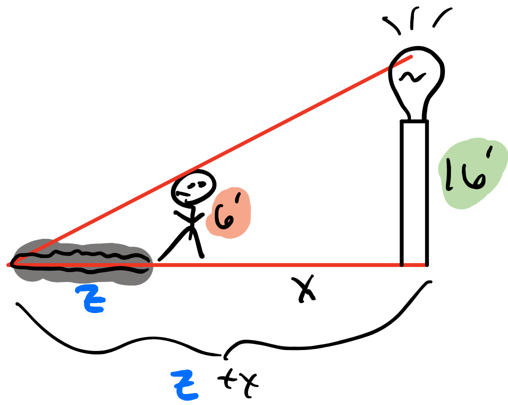
$$\frac{d\theta}{dt} = \frac{\cancel{2}^{\,1}}{\cancel{25}} \cdot \left(\frac{\cancel{-25}^{\,-1}}{\cancel{24}_{\,12}}\right)$$

$$\frac{d\theta}{dt} = \frac{-1}{12} \text{ RADIANS/second}$$

Unit 4.5.3 RIGHT TRIANGLES Solving Related Rates Problems

Right Triangles – Similar Triangles

2. A man 6 feet tall walks at a rate of 5 feet per second toward a street light that is 16 feet tall. At what rate is the length of his shadow changing when he is 10 feet from the base of the light?



$\dfrac{dx}{dt} = \dfrac{-5 \text{ ft}}{1 \text{ sec}}$

x = distance man is from light
z = shadow length

$\dfrac{z}{6} = \dfrac{z + x}{16}$

$16z = 6z + 6x$

$10z = 6x$

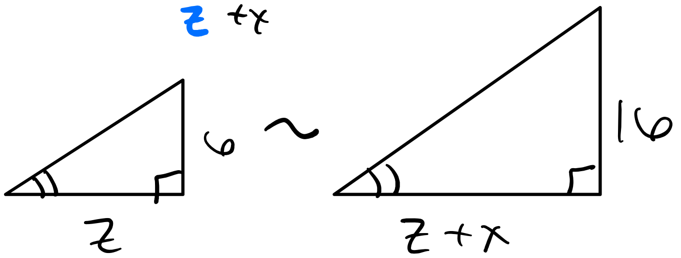
$\dfrac{d}{dt} 10z = \dfrac{d}{dt} 6x$

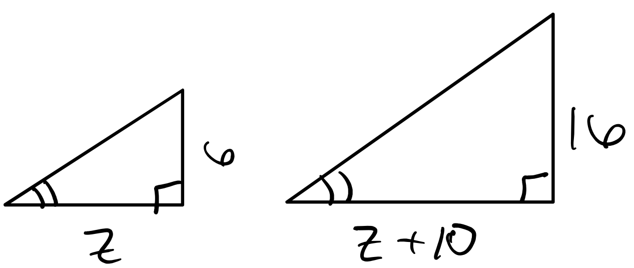
$10 \cdot \dfrac{dz}{dt} = 6 \cdot \dfrac{dx}{dt}$

$\dfrac{dz}{dt} = \dfrac{3}{5} \dfrac{dx}{dt}$

$\dfrac{dz}{dt} = \dfrac{3}{5}(-5)$

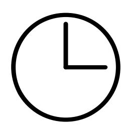
$\dfrac{dz}{dt} = -3$ feet/sec





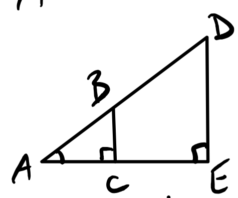
Find $\dfrac{dz}{dt}$

$x = 10$



Geometry Review

AA Similarity



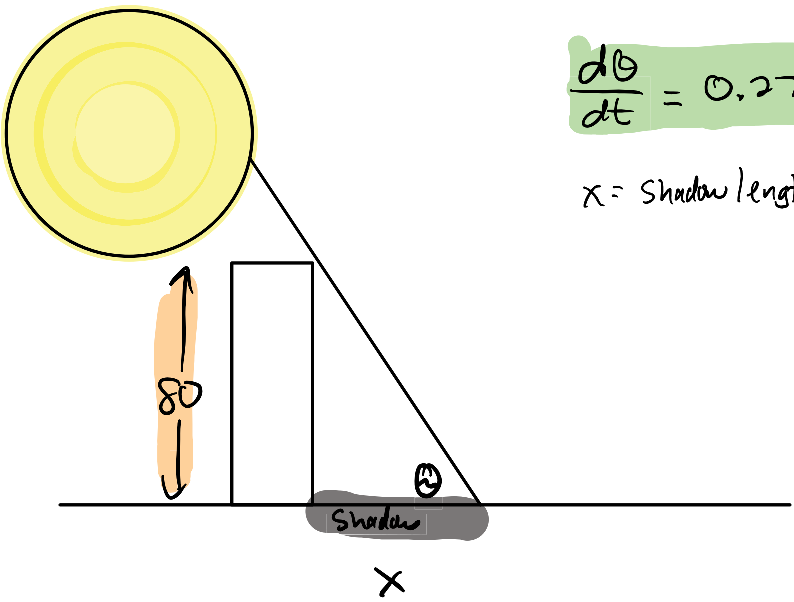
$\Delta ABC \sim \Delta ADE$

$\therefore \dfrac{AC}{AE} = \dfrac{BC}{DE}$

Unit 4.5.3 RIGHT TRIANGLES Solving Related Rates Problems

Right Triangles - Shadows

3. On a morning when the sun will pass directly overhead, the shadow of an 80—foot building on level ground is 60 feet long. At the moment in question, the angle ϑ the sun makes with the ground is increasing at the rate of 0.27 radians per minute. At what rate is the length of the shadow decreasing?



$\frac{d\theta}{dt} = 0.27$ RAD/MIN

x = shadow length

$\tan\theta = \frac{80}{x}$

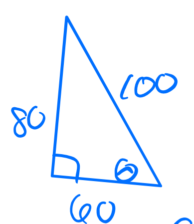
$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(80x^{-1})$

$\sec^2\theta \cdot \frac{d\theta}{dt} = -80x^{-2}\frac{dx}{dt}$

$\sec^2\theta \cdot (0.27) = \frac{-80}{x^2}\frac{dx}{dt}$

Find $\frac{dx}{dt}$

$x = 60$



$\cos\theta = \frac{60}{100}$

$\sec\theta = \frac{100}{60}$

$\sec\theta = 5/3$

$\sec^2\theta = \frac{25}{9}$

PT
3-4-5
6-8-10
60-80-100

$\left(\frac{25}{9}\right)(0.27) = \frac{-80}{(60)^2}\frac{dx}{dt}$

$\frac{dx}{dt} = \left(\frac{25}{9}(0.27)\cdot\frac{60^2}{-80}\right)$ ft/min

$\frac{dx}{dt} = -33.75$ ft/min