# Unit 4.6 Approximating Values of a Function Using Local Linearity and Linearization

## TOPIC QUESTION 5

| $x$ | 2.8 | 3.0 | 3.2 | 3.4 |
|---|---|---|---|---|
| $g'(x)$ | 1.05 | −1.2 | −0.8 | 1.3 |

Selected values of the derivative of the function $g$ are given in the table above. It is known that $g(3) = 17$. What is the approximation for $g(3.2)$ found using the line tangent to the graph of $g$ at $x = 3$?

(A) 16.76

(B) 16.80

(C) 16.84

(D) 17.40

$y - g(3) = g'(3)[x - 3]$

$y - 17 = -1.2(x - 3)$

$y = -1.2(x-3) + 17$

$g(3.2) \approx -1.2(3.2-3) + 17$

$\approx -1.2(0.2) + 17$

$\approx -0.24 + 17$

$g(3.2) \approx 16.76$

## TOPIC QUESTION 6

PoT SoT $f'' > 0$

Let $f$ be a differentiable function such that $f(4) = 7$ and $f'(4) = \frac{1}{5}$. The graph of $f$ is concave up on the interval $(3, 5)$. Which of the following is true about the approximation for $f(3.5)$ found using the line tangent to the graph of $f$ at $x = 4$?

(A) $f(3.5) \approx 6.9$ and this approximation is an overestimate of the value of $f(3.5)$. T

(B) $f(3.5) \approx 6.9$ and this approximation is an underestimate of the value of $f(3.5)$. T True

(C) $f(3.5) \approx 7.1$ and this approximation is an overestimate of the value of $f(3.5)$. F

(D) $f(3.5) \approx 7.1$ and this approximation is an underestimate of the value of $f(3.5)$. F True

$y - 7 = \frac{1}{5}(x-4)$

$y = \frac{1}{5}(x-4) + 7$

$f(3.5) \approx \frac{1}{5}(3.5-4) + 7$

$\approx \frac{1}{5}(-0.5) + 7$

$\approx -0.1 + 7$

$f(3.5) \approx 6.9$

## TOPIC QUESTION 7 CALCULATOR

$f'' > 0$

Let $f$ be a function such that at each point $(x, y)$ on the graph of $f$, the slope is given by $\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{4}y^2$. The graph of $f$ passes through the point $(1, -2)$ and is concave up on the interval $1 < x < 1.5$. Let $k$ be the approximation for $f(1.3)$ found by using the locally linear approximation of $f$ at $x = 1$. Which of the following statements about $k$ is true?

PoT $(1, -2)$ SoT $\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{1}{2}(1) - \frac{1}{4}(-2)^2 = \frac{1}{2} - 1 = -\frac{1}{2}$

(A) $k = -2.65$ and is an underestimate for $f(1.3)$. T

(B) $k = -2.65$ and is an overestimate for $f(1.3)$.

(C) $k = -2.15$ and is an underestimate for $f(1.3)$. T T

(D) $k = -2.15$ and is an overestimate for $f(1.3)$. T

$y + 2 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}(x-1) - 2$

$f(1.3) \approx -\frac{1}{2}(1.3-1) - 2$

$\approx -2.15$

$f'' < 0$

## Topic Question 8

Let $f$ be a function such that at each point $(x, y)$ on the graph of $f$, the slope is given by $\frac{dy}{dx} = y^2 - x$. The graph of $f$ passes through the point $(1, 2)$ and is concave down on the interval $1 < x < 1.5$. Let $k$ be the approximation for $f(1.2)$ found by using the locally linear approximation of $f$ at $x = 1$. Which of the following statements about $k$ is true?

PoT SoT

$(1,2)$ $\left.\frac{dy}{dx}\right|_{(1,2)} = 2^2 - 1 = 3$

Ⓐ $k = 5.6$ and is an overestimate for $f(1.2)$.

Ⓑ $k = 5.6$ and is an underestimate for $f(1.2)$.

Ⓒ $k = 2.6$ and is an overestimate for $f(1.2)$.

Ⓓ $k = 2.6$ and is an underestimate for $f(1.2)$.

$y - 2 = 3(x-1)$

$y = 3(x-1) + 2$

$f(1.2) = 3(1.2-1) + 2$

$= 3(0.2) + 2$

$= 0.6 + 2$

$= 2.6$

## Topic Question 9

| $x$ | 3.8 | 4.0 | 4.2 | 4.4 |
|---|---|---|---|---|
| $g'(x)$ | $-0.8$ | $2.2$ | $1.8$ | $-1.2$ |

SoT PoT

Selected values of the derivative of the function $g$ are given in the table above. It is known that $g(4) = 12$. What is the approximation for $g(4.2)$ found using the line tangent to the graph of $g$ at $x = 4$?

Ⓐ 12.44

Ⓑ 12.40

Ⓒ 12.36

Ⓓ 11.60

$y - 12 = 2.2(x-4)$

$y = 2.2(x-4) + 12$

$g(4.2) \approx 2.2(4.2-4) + 12$

$\approx 2.2(0.2) + 12$

$\approx 0.44 + 12$

$g(4.2) \approx 12.44$

$f'' < 0$

## Topic Question 10

PoT SoT

Let $g$ be a differentiable function such that $g(3) = 2$ and $g'(3) = -\frac{3}{4}$. The graph of $g$ is concave down on the interval $(2, 4)$. Which of the following is true about the approximation for $g(2.6)$ found using the line tangent to the graph of $g$ at $x = 3$?

Ⓐ $g(2.6) \approx 1.7$ and this approximation is an overestimate of the value of $g(2.6)$.

Ⓑ $g(2.6) \approx 1.7$ and this approximation is an underestimate of the value of $g(2.6)$.

Ⓒ $g(2.6) \approx 2.3$ and this approximation is an overestimate of the value of $g(2.6)$.

Ⓓ $g(2.6) \approx 2.3$ and this approximation is an underestimate of the value of $g(2.6)$.

$y - 2 = -\frac{3}{4}(x-3)$

$y = -\frac{3}{4}(x-3) + 2$

$y = -\frac{3}{4}(2.6-3.0) + 2$

$y = -\frac{3}{4}(-0.4) + 2$

$y = +.3 + 2$ $\quad -\frac{3}{4} \cdot \left(-\frac{4}{10}\right) = 0.3$

$y = 2.3$

# Unit 4.6 Approximating Values of a Function Using Local Linearity and Linearization

## APCLASSROOM FRQ 2

Let $h$ be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of $h$ is given by $h'(x) = \frac{x^2-2}{x}$ for all $x \neq 0$.

Does the line tangent to the graph of $h$ at $x = 4$ lie above or below the graph of $h$ for $x > 4$? Why?

$$h'(x) = \frac{x^2-2}{x} = \frac{x^2}{x} - \frac{2}{x} = x - 2x^{-1}$$

$$h''(x) = 1 + 2x^{-2}$$

$$h''(x) = 1 + \frac{2}{x^2} = \frac{x^2+2}{x^2}$$

$$h''(x) = \frac{x^2}{x^2} + \frac{2}{x^2}$$

$$h''(x) = \frac{x^2+2}{x^2}$$

Pos

4

$\therefore h''(x) > 0$ for $x > 4$

$\therefore h(x)$ is concave up there

1 point

$\therefore$ tangent line is below graph of $h$.

## APCLASSROOM FRQ 5

POT

| $x$ | $-1.5$ | $-1.0$ | $-0.5$ | $0$ | $0.5$ | $1.0$ | $1.5$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-1$ | $-4$ | $-6$ | $-7$ | $-6$ | $-4$ | $-1$ |
| $f'(x)$ | $-7$ | $-5$ | $-3$ | $0$ | $3$ | $5$ | $7$ |

SOT

Let $f$ be a function that is differentiable for all real numbers. The table above gives the values of $f$ and its derivative $f'$ for selected points $x$ in the closed interval $-1.5 \le x \le 1.5$. The second derivative of $f$ has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

Concave up

Write an equation of the line tangent to the graph of $f$ at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

$$\boxed{y + 4 = 5(x-1)} \leftarrow \text{1 pt}$$

$$y = 5(x-1) - 4$$

$$f(1.2) \approx 5(1.2-1) - 4$$

$$\approx 5(0.2) - 4$$

$$5 \cdot \tfrac{2}{10} = 1$$

$$\approx 1 - 4$$

$$\text{1 pt} \rightarrow \boxed{f(1.2) \approx -3}$$

$f$ is concave up on $[-1.5, 1.5]$

$\therefore f(1.2) \approx -3$ is less than the actual value of $f(1.2)$

← 1 pt

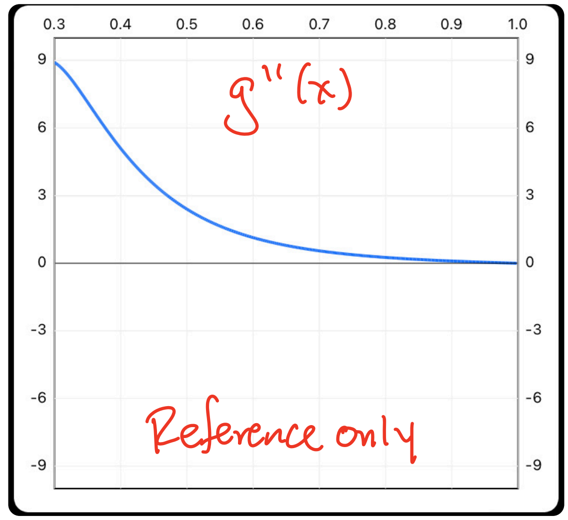
## APClassroom FRQ 9 Calculator

The function $g$ is defined for $x>0$ with $g(1)=2$, $g'(x)= \sin(x+1/x)$, and $g''(x)=(1-1/x^2)\cos(x+1/x)$.

Does the line tangent to the graph of $g$ at $x=0.3$ lie above or below the graph of $g$ for $0.3<x<1$? Why?

$g''(x) > 0$ on $0.3 < x < 1$

$\therefore$ tangent line lies below $g(x)$ on this interval.




## APClassroom MC 14 Calculator

POT SOT

A differentiable function $f$ has the property that $f(5)=3$ and $f'(5)=4$. What is the estimate for $f(4.8)$ using the local linear approximation for $f$ at x= 5 ?

(A) 2.2

(B) 2.8

(C) 3.4

(D) 3.8

(E) 4.6

$y-3=4(x-5)$

$y=4(x-5)+3$

$f(4.8)=4(4.8-5)+3$

$=4(-0.2)+3$

$=-0.8+3$

$f(4.8)=2.2$

# Unit 4.6 Approximating Values of a Function Using Local Linearity and Linearization

APCLASSROOM MC 15

| $x$ | 1 | 1.1 | 1.2 | 1.3 | 1.4 |
|---|---|---|---|---|---|
| $f'(x)$ | 8 | 10 | 12 | 13 | 14.5 |

SOT

POT

Concave up

The function $f$ is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of $f'$, are given for selected values of $x$ in the table above.

Write an equation for the line tangent to the graph of $f$ at $x = 1$. Use this line to approximate $f(1.4)$.

1pt $\boxed{y - 15 = 8(x-1)}$

$f(1.4) \approx 8(1.4-1) + 15$

$\approx 8(0.4) + 15$

$\approx 3.2 + 15$

$\boxed{f(1.4) \approx 18.2}$ 1 pt

APCLASSROOM MC 78

POT SOT Concave up

The function $f$ is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of $f$ at $x = 2$?

(A) 0.4

(B) 0.6

(C) 0.7

(D) 1.3

(E) 1.4

$y - 1 = 4(x-2)$

$y - 1 = 4(1.9-2)$

$y = 4(-0.1) + 1$

$y = -0.4 + 1$

$y = 0.6$