Unit 4.7 Using L'Hospital's Rule for Finding Limits of Indeterminate Form

Topic Question 2

Let f be the function defined by $f(x) = 2x + 3e^{-5x}$, and let g be a differentiable function with derivative given by $g'(x) = \frac{1}{x} + 4\cos\left(\frac{5}{x}\right)$. It is known that $\lim_{x\to\infty} g(x) = \infty$. The value of $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is

$\lim_{x\to\infty} \frac{f(x)}{g(x)}$ produces $\frac{\infty}{\infty}$

$\lim_{x\to\infty} f(x) = 2(\infty) + \underbrace{\frac{3}{e^{5\infty}}}_{0} = \infty$

$\lim_{x\to\infty} \frac{2 + 3e^{-5x}(-5)}{\frac{1}{x} + 4\cos\left(\frac{5}{x}\right)} = \frac{2+0}{0+4(1)} = \frac{2}{4} = \frac{1}{2}$

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) nonexistent

Topic Question 3 Show the work as if this is a FRQ

$\lim_{x\to 0} \frac{6e^{4x} - 2e^{3x} - 4}{\sin(2x)} = \lim_{x\to 0} \frac{24e^{4x} - 6e^{x}}{2\cos(2x)} = \frac{24e^{4\cdot 0} - 6e^{0}}{2\cos(2\cdot 0)} = \frac{24\cdot 1 - 6\cdot 1}{2\cdot 1} = \frac{18}{2} = 9$

Apply L'Hospital's

(A) 2

$\lim_{x\to 0} (6e^{4x} - 2e^{3x} - 4) = 6\cdot e^{4\cdot 0} - 2e^{3\cdot 0} - 4 = 0$

$\lim_{x\to 0} \sin(2x) = \sin(2\cdot 0) = 0$

This limit produces indeterminant form 0/0

(B) 4

(C) 9

(D) 18

Topic Question 4

$\lim_{x\to\pi} \frac{\overbrace{x + \pi\sec x}^{\pi - \pi = 0}}{\underbrace{x^2 - \pi^2}_{0}}$ is

L'Hospital's

$= \lim_{x\to\pi} \frac{1 + \pi\sec x\tan x}{2x} = \frac{1 + \pi\overbrace{\sec(\pi)}^{-1}\cdot\overbrace{\tan(\pi)}^{0}}{2(\pi)} = \frac{1}{2\pi}$

(A) $-\frac{\pi}{2}$

(B) 0

(C) $\frac{1}{2\pi}$

(D) nonexistent

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TOPIC QUESTION 5

Let f be the function defined by $f(x) = 3x + 2e^{-3x}$, and let g be a differentiable function with derivative given by $g'(x) = 4 + \frac{1}{x}$. It is known that $\lim_{x\to\infty} g(x) = \infty$. The value of $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is

$\to \infty$

$\lim_{x\to\infty}(3x + 2e^{-3x}) = \infty + 2\cdot 0 = \infty$

L'HOSPITALS

$\lim_{x\to\infty} \frac{3 - 6e^{-3x}}{4 + \frac{1}{x}} = \frac{3 - 6\cdot 0}{4 + 0} = \frac{3}{4}$

(A) 0

(B) $\frac{3}{4}$

(C) 1

(D) nonexistent

TOPIC QUESTION 6 SHOW THE WORK AS IF THIS IS A FRQ

$\lim_{t\to 0} \frac{\sin t}{\ln(2e^t - 1)} = \lim_{t\to 0} \frac{\cos t}{\frac{1}{2e^t - 1}\cdot 2e^t} = \frac{\cos 0}{\frac{1}{2e^0 - 1}\, 2e^0} = \frac{1}{\frac{1}{2\cdot 1 - 1}\cdot 2\cdot 1} = \frac{1}{1\cdot 2} = \frac{1}{2}$

L'HOSPITAL'S Rule

(A) -1

$\lim_{t\to 0}(\sin t) = \sin(0) = 0$

$\lim_{t\to 0}[\ln(2e^t - 1)] = \ln(2e^0 - 1) = \ln(2 - 1) = \ln 1 = 0$

This limit produces indeterminant form 0/0

(B) 0

(C) $\frac{1}{2}$

(D) 1

TOPIC QUESTION 7

$\lim_{x\to\frac{\pi}{2}} \frac{3\cos x}{2x - \pi}$ is $= \lim_{x\to \pi/2} \frac{-3\sin x}{2} = \frac{-3\sin(\frac{\pi}{2})}{2} = \frac{-3\cdot 1}{2}$

L'HOSPITALS

(A) $-\frac{3}{2}$

(B) 0

(C) $\frac{3}{2}$

(D) nonexistent

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APCLASSROOM FRQ 52

The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

g is continuous, $\therefore \lim_{x \to 0} g(x) = g(0)$

$$\lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{-2\sin(2x)}{2x} = \lim_{x \to 0} \frac{-4\cos(2x)}{2} = \frac{-4\cos(2 \cdot 0)}{2} = \frac{-4 \cdot 1}{2} = -2$$

L'HOSPITAL'S Rule

L'HOSPITAL'S Rule

$\lim_{x \to 0} [\cos(2x) - 1] = 0$

$\lim_{x \to 0} (x^2) = 0$

This limit produces indeterminate form 0/0

$\lim_{x \to 0} [-2\sin(2x)] = 0$

$\lim_{x \to 0} (2x) = 0$

This limit produces indeterminate form 0/0

APCLASSROOM MC 64

$$\lim_{h \to 0} \frac{e^{(2+h)} - e^2}{h} = \lim_{h \to 0} \frac{e^{(2+h)} \cdot 1}{1} = e^{2+0}$$

0/0

L'HOSPITALS

Ⓐ 0

Ⓑ 1

Ⓒ $2e$

Ⓓ e^2

Ⓔ $2e^2$

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APCLASSROOM MC 66

$\lim_{x \to 0} \frac{\sin x \cos x}{x}$ is $= \lim_{x \to 0} \frac{\cos x \cdot \cos x - \sin x \cdot \sin x}{1} = \cos^2(0) - \sin^2(0)$

L'HOSPITAL

$= 1^2 - 0^2$

$= 1$

(A) −1

(B) 0

(C) 1

(D) $\frac{\pi}{4}$

(E) nonexistent

APCLASSROOM MC 67

$\lim_{x \to 2} \frac{x^2+x-6}{x^2-4}$ is $= \lim_{x \to 2} \frac{2x+1}{2x} = \frac{5}{4}$

0

0

L'HOSPITAL

(A) $-\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{5}{4}$

(E) nonexistent