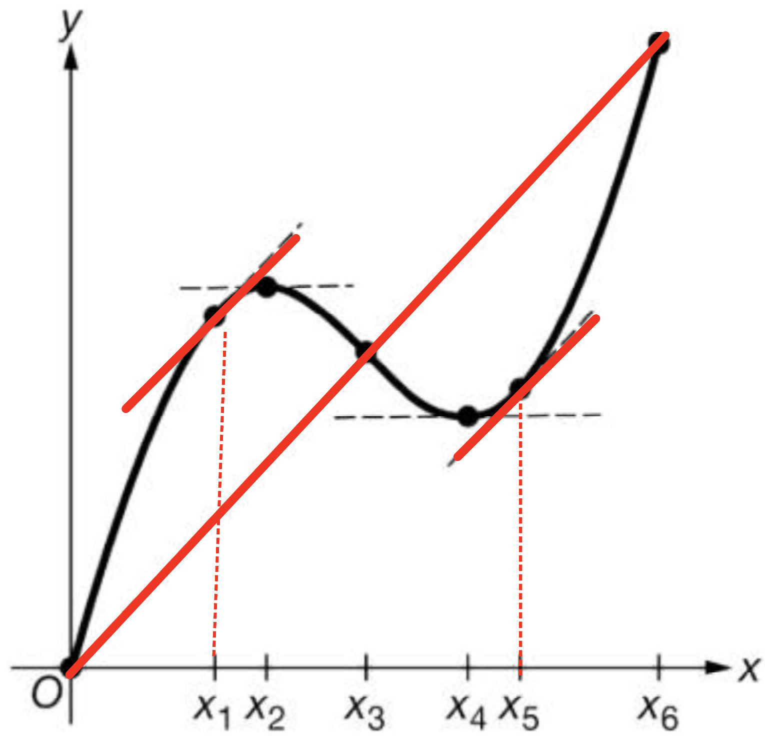
Unit 5.1 Using the Mean Value Theorem

1.



Graph of g

MVT

The function g shown in the figure above is continuous on the closed interval $[0, x_6]$ and differentiable on the open interval $(0, x_6)$, where x_1, x_2, x_3, x_4, x_5, and x_6 are points on the x-axis. Based on the graph, what are all values of x that satisfy the conclusion of the Mean Value Theorem applied to g on the closed interval $[0, x_6]$?

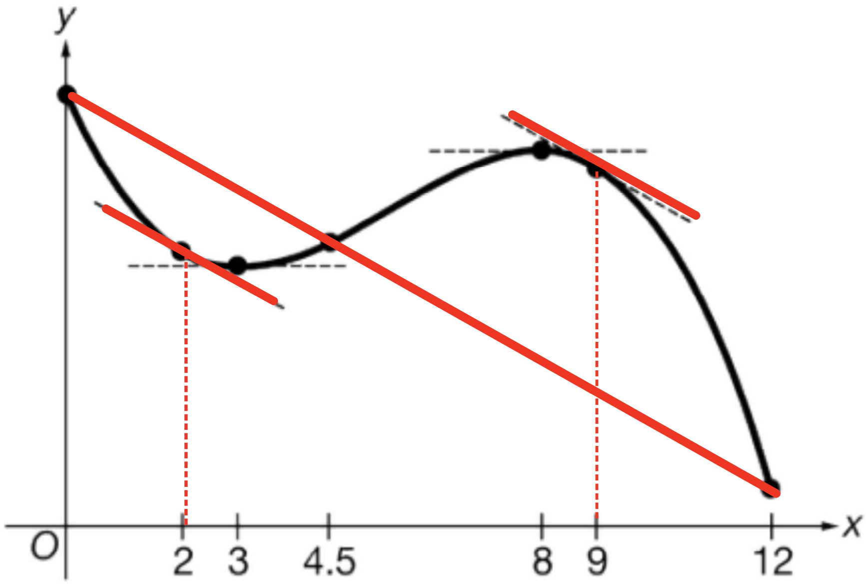
(A) x_3 only, because this is the value where $g(x)$ equals the average rate of change of g on $[0, x_6]$.

(B) x_2 and x_4 only, because these are the values where $g'(x) = 0$ on $[0, x_6]$.

(C) x_1 and x_5 only, because these are the values where the instantaneous rate of change of g at those values is equal to the average rate of change of g on $[0, x_6]$.

(D) x_1, x_3, and x_5 only, because these are the values where either the instantaneous rate of change of g at the value is equal to the average rate of change of g on $[0, x_6]$ or the value of $g(x)$ is equal to the average rate of change of g on $[0, x_6]$.

2.



The function f shown in the figure above is continuous on the closed interval $[0, 12]$ and differentiable on the open interval $(0, 12)$. Based on the graph, what are all values of x that satisfy the conclusion of the Mean Value Theorem applied to f on the closed interval $[0, 12]$?

(A) 4.5 only because this is the value where $f(x)$ equals the average rate of change of f on $[0, 12]$.

(B) 3 and 8 because these are the values where $f'(x) = 0$ on $[0, 12]$.

(C) 2 and 9 only because these are the values where the instantaneous rate of change of f at those values is equal to the average rate of change of f on $[0, 12]$.

(D) 2, 4.5, and 9 because these are the values where either the instantaneous rate of change of f at the value is equal to the average rate of change of f on $[0, 12]$ or the value of $f(x)$ is equal to the average rate of change of f on $[0, 12]$.

3. The Mean Value Theorem can be applied to which of the following functions on the closed interval $[-5, 5]$?

(A) $f(x) = \frac{1}{\sin x}$ $x \neq -2\pi, 0, 2\pi$

(B) $f(x) = \frac{x-1}{|x-1|}$ $x \neq 1$

(C) $f(x) = \frac{x^2}{x^2-36}$ ✓ $x \neq -6, x \neq 6$

(D) $f(x) = \frac{x^2}{x^2-4}$ $x \neq -2, x \neq 2$

4. The Mean Value Theorem can be applied to which of the following functions on the closed interval $[-3, 3]$?

(A) $f(x) = x^{\frac{2}{3}}$ X Not diff at $x=0$

(B) $f(x) = |x - 1|$ X Not diff at $x=1$

(C) $f(x) = \frac{x-2}{x-5}$ ✓ $x \neq 5$

(D) $f(x) = \frac{x-5}{x-2}$ $x \neq 2$

m = 4
(2,5) to (5,17)
(2,5) to (11,41)

m=4 m=4 m=11/2 m=13/4

5.

x	0	2	5	7	11
$f(x)$	13	5	17	28	41

36/9 23/5 m = 4/5 15/7 28/11

Selected values of a differentiable function f are given in the table above. What is the fewest possible number of values of c in the interval $[0, 11]$ for which the Mean Value Theorem guarantees that $f'(c) = 4$?

(A) Zero
(B) One
(C) Two
(D) Three

m = 6/2 m = 12/2 m = 11/2 m = 13/2

6.

x	1	3	5	7	9
$f(x)$	0	6	18	29	42

36/6 23/4 18/4 29/6 42/8 x

Selected values of a differentiable function f are given in the table above. What is the fewest possible number of values of c in the interval $[1, 9]$ for which the Mean Value Theorem guarantees that $f'(c) = 6$?

(A) Zero
(B) One
(C) Two
(D) Three

7. The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$ for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$
(B) For $-2 < k < 2$, $f'(k) < 0$
(C) For $-2 < k < 2$, $f'(k)$ exists.
(D) For $-2 < k < 2$, $f'(k)$ exists, but f is not continuous.
(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

Diff?

8.

x	$f(x)$
1	2.4
3	3.6
5	5.4

$m = 0.6$

$m = 0.9$

we need $0.6 < f'(3) < 0.9$ b/c $f'' > 0$

The table above gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

(A) 0.6
(B) 0.7
(C) 0.9
(D) 1.2
(E) 1.5

9. A differentiable function f has the property that $f'(x) \leq 3$ for $1 \leq x \leq 8$ and $f(5) = 6$. Which of the following could be true?

I. $f(2) = 0$ ✓

II. $f(6) = -2$ ✓

III. $f(7) = 13$ ✗

$(2,0), (5,6) \Rightarrow m = 2$

$(5,6), (6,-2) \Rightarrow m = \frac{8}{-1} = -8$

$(5,6), (7,13) \Rightarrow m = \frac{7}{2} > 3$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

Cont & Diff

10. Let f be the function given by $f(x) = x^3 - 2x^2 + 5x - 16$. For what value of x in the closed interval [0,5] does the instantaneous rate of change of f equal the average rate of change of f over that interval?

(A) 0
(B) 5/3
(C) 5/2
(D) 3
(E) 5

ARC

$f(0) = -16$

$f(5) = 125 - 50 + 25 - 16$

$f(5) = 84$

$ARC = \frac{84+16}{5-0} = \frac{100}{5} = 20$

$f' = 3x^2 - 4x + 5$

MVT

$20 = 3x^2 - 4x + 5$

$0 = 3x^2 - 4x - 15$

$0 = 3x^2 - 9x + 5x - 15$

$0 = 3x(x-3) + 5(x-3)$

$0 = (x-3)(3x+5)$

$x = 3$, $\cancel{x = -5/3}$

$m = -45x^2$
$Add = -4x$
$T = -9x, 5x$

diff → Cont

11. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x *between a and b*?

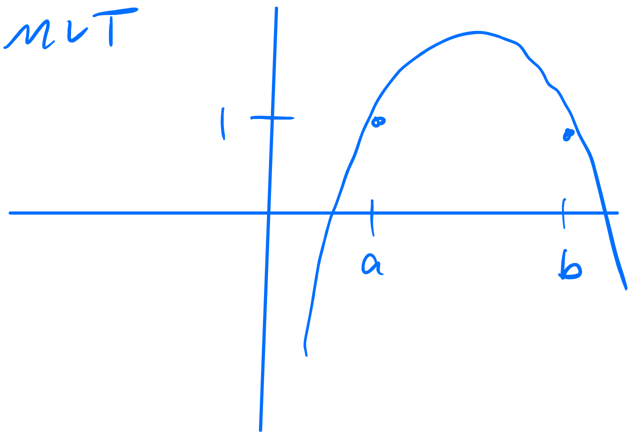
I. $f(x) = 0$

II. $f'(x) = 0$

III. $f''(x) = 0$

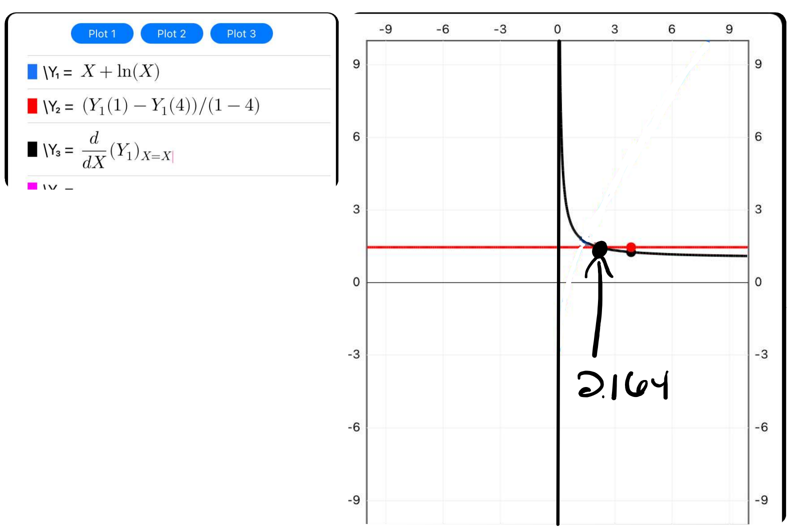
(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

MVT



12. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1,4] ?

(A) 0.456
(B) 1.244
(C) 2.164
(D) 2.342
(E) 2.452



FRQ 1

x	−1.5	−1.0	−0.5	0	0.5	1.0	1.5
$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

$\therefore f'$ is differentiable

13. Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

f' is differentiable on $-1.5 \le x \le 1.5$

$\underbrace{\text{By MVT there exists a } c \text{ on } (0, 0.5) \text{ such that}}_{+1}$ $\underbrace{f''(c) = \frac{f(0) - f(0.5)}{0 - 0.5} = \frac{0-3}{0.5} = \frac{-3}{1/2} = -6}_{+1}$

$\therefore r = -6$

FRQ 2

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

−7 −13/4 −8 −7

−6 −4 −1

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (° C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

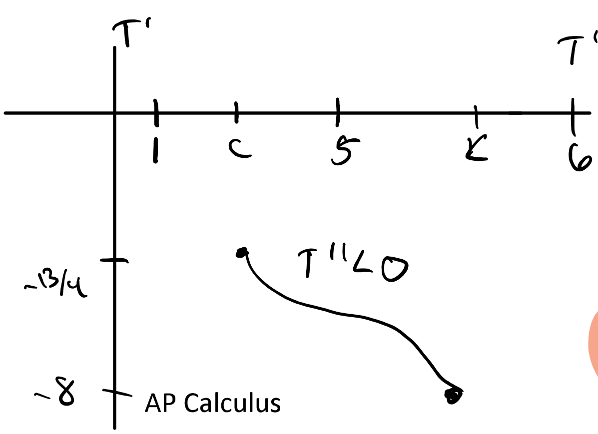
Continuous

14. Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

$ARC = \frac{f(1) - f(5)}{1-5} = \frac{93-70}{-4} = \frac{13}{-4}$ $\therefore T'(c) = \frac{-13}{4}$ for some c on $(1,5)$

$ARC = \frac{f(5) - f(6)}{5-6} = \frac{70-62}{-1} = \frac{8}{-1} = -8$ $\therefore T'(k) = -8$ for some k on $(5,6)$

By MVT

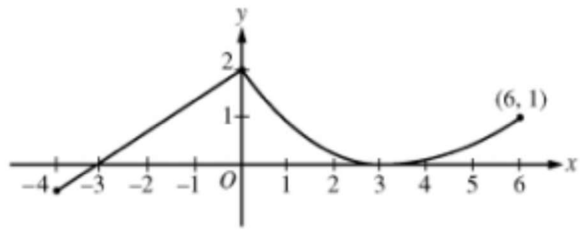


T' is decreasing somewhere between $x = c$ and $x = k$

$\therefore T'' < 0$ somewhere between $x = c$ and $x = k$

OUCH

FRQ 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

15. Is there a value of $a, -4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

+1

- $f(x)$ is continuous on $[3,6]$ and differentiable on $(3,6)$

$$ARC = \frac{f(3) - f(6)}{3-6} = \frac{0-1}{3-6} = \frac{-1}{-3} = \frac{1}{3}$$

+1 By MVT there exists a c on $(3,6)$ such that $f'(c) = 1/3$