Unit 5.2 Extreme Value Theorem

APClassroom Hw 5.2 MC

which is continuous on $[0, 2\pi]$?

1. Which of the following functions of x is guaranteed by the Extreme Value Theorem to have an absolute maximum on the interval $[0, 2\pi]$?

(A) $y = \frac{1}{1+\sin x}$ $\sin x \neq -1 \Rightarrow x \neq \frac{3\pi}{2}$ Not Continuous

(B) $y = \frac{1}{x^2+\pi}$ $x^2+\pi \neq 0 \Rightarrow x^2 \neq -\pi$ no real solution ∴ continuous

(C) $y = \frac{x^2-2\pi x+\pi^2}{x-\pi}$ $x-\pi \neq 0 \Rightarrow x \neq \pi$ Not Continuous

(D) $y = \frac{|x-\pi|}{x-\pi}$ $x-\pi \neq 0 \Rightarrow x \neq \pi$ Not Continuous

2. Which of the following functions of x is guaranteed by the Extreme Value Theorem to have an absolute maximum on the interval $[0, 4]$? which is continuous on $[0, 4]$?

~~(A)~~ $y = \tan x$ $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

(B) $y = \tan^{-1} x$ Continuous

(C) $y = \frac{x^2-16}{x^2+x-20}$ $(x+5)(x-4) \neq 0$ ∴ $x \neq 4$, $\cancel{x \neq -5}$

(D) $y = \frac{1}{e^x-1}$ $e^x - 1 \neq 0 \Rightarrow e^x \neq 1 \Rightarrow x \neq 0$

3. Let g be the function given by $g(x) = \sqrt{1-\sin^2 x}$. Which of the following statements could be false on the interval $0 \leq x \leq \pi$? Continuous

✓ Cont. (A) By the Extreme Value Theorem, there is a value c such that $g(c) \leq g(x)$ for $0 \leq x \leq \pi$. (ABS MIN)

✓ Cont. (B) By the Extreme Value Theorem, there is a value c such that $g(c) \geq g(x)$ for $0 \leq x \leq \pi$. ABS MIN

✓ cont. (C) By the Intermediate Value Theorem, there is a value c such that $g(c) = \frac{g(0)+g(\pi)}{2}$.

✓ cont & Diff ✗ (D) By the Mean Value Theorem, there is a value c such that $g'(c) = \frac{g(\pi)-g(0)}{\pi-0}$.

$g'(x) = \frac{-\sin x \cos x}{\sqrt{1-\sin^2 x}}$

$1-\sin^2 x \neq 0$

$1 \neq \sin^2 x$

$1 \neq \sin x$

$x \neq \pi/2$

∴ $g'(x)$ is not diff at $x = \pi/2$

4. Let g be the function given by $g(x) = \sqrt{1+\cos x}$. Which of the following statements could be false on the interval $\frac{\pi}{2} \le x \le \frac{7\pi}{4}$?

Cont. ✓ (A) By the Extreme Value Theorem, there is a value c such that $g(c) \le g(x)$ for $\frac{\pi}{2} \le x \le \frac{7\pi}{4}$.

Cont ✓ (B) By the Extreme Value Theorem, there is a value c such that $g(c) \ge g(x)$ for $\frac{\pi}{2} \le x \le \frac{7\pi}{4}$.

cont. ✓ (C) By the Intermediate Value Theorem, there is a value c such that $g(c) = \frac{g(\frac{\pi}{2})+g(\frac{7\pi}{4})}{2}$.

cont ✓ & Diff ✗ (D) By the Mean Value Theorem, there is a value c such that $g'(c) = \frac{g(\frac{7\pi}{4})-g(\frac{\pi}{2})}{\frac{7\pi}{4}-\frac{\pi}{2}}$.

$g' = \frac{-\sin x}{2\sqrt{1+\cos x}}$

g' is undefined when $2\sqrt{1+\cos x} = 0$

$1+\cos x = 0$

$\cos x = -1$

$x = \pi$

$\therefore g'$ is not diff at $x = \pi$

5.

x	0	1	2	3
$f(x)$	0	4	7	6

Let f be a function with selected values given in the table above. Which of the following statements must be true?

continuous ✗ I. By the Intermediate Value Theorem, there is a value c in the interval $(0,3)$ such that $f(c) = 2$.
cont ✗ & diff ✗ II. By the Mean Value Theorem, there is a value c in the interval $(0,3)$ such that $f'(c) = 2$.
continuous ✗ III. By the Extreme Value Theorem, there is a value c in the interval $[0,3]$ such that $f(c) \le f(x)$ for all x in the interval $[0,3]$.

(A) None
(B) I only
(C) II only
(D) I, II, and III

6.

x	0	1	2	3
$f(x)$	15	14	12	9

Let f be a function with selected values given in the table above. Which of the following statements must be true?

continuous ✗ I. By the Intermediate Value Theorem, there is a value c in the interval $(0,3)$ such that $f(c) = 10$.
cont ✗ & diff ✗ II. By the Mean Value Theorem, there is a value c in the interval $(0,3)$ such that $f'(c) = -2$.
continuous ✗ III. By the Extreme Value Theorem, there is a value c in the interval $[0,3]$ such that $f(c) \le f(x)$ for all x in the interval $[0,3]$.

(A) None
(B) I only
(C) II only
(D) I, II, and III

7.

x	10	11	12	13	14
$f(x)$	5	2	3	6	5

Dec INC Dec

The table above gives values of the continuous function f at selected values of x. If f has exactly two critical points on the open interval (10, 14), which of the following must be true?

(A) $f(x) > 0$ for all x in the open interval (10, 14). Some y-values could be neg

(B) $f'(x)$ exists for all x in the open interval (10, 14). Diff is not guaranteed.

(C) $f'(x) < 0$ for all x in the open interval (10, 11). f is increasing somewhere between $11 < x < 13$

(D) $f(12) \neq 0$ This would require more CV if $f'(12) = 0$

8. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

$f' = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

$f' = 0$: $1 - \ln x = 0$, $1 = \ln x$, $x = e$

f' undefined: $x^2 = 0$, $x = 0$

x	$f(x)$
e	$\frac{\ln e}{e} = \frac{1}{e}$
0	und

9. If f is continuous for $a \leq x \leq b$ and differentiable for $a<x<b$, which of the following could be false?

(A) $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c such that $a<c<b$. MVT True

(B) $f'(c)=0$ for some c such that $a<c<b$.

(C) f has a minimum value on $a \leq x \leq b$. EVT True

(D) f has a maximum value on $a \leq x \leq b$. EVT True

10. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3

(B) $-\frac{3}{2}$

(C) $-\frac{1}{3}$

(D) 0

(E) There is no such k.

$g' = 2x \cdot e^{kx} + x^2 \cdot e^{kx} \cdot k$

~~$g' = 0$~~

~~g' is undef none~~

$0 = xe^{kx}(2 + kx)$

at $x = 2/3$ $\quad 0 = \frac{2}{3}e^{2/3k}$ no solution

$0 = 2 + \frac{2}{3}k$

$-2 = \frac{2}{3}k$

$-3 = k$

11. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One

(B) Two

(C) Three

(D) Five

(E) Nine

$f' = 5(x+2)^4 \cdot 1 (x-3)^4 + (x+2)^5 \cdot 4(x-3)^3 \cdot 1$

$f' = (x+2)^4(x-3)^3\left[5(x-3) + (x+2)\cdot 4\right]$

$f' = (x+2)^4(x-3)^3\left[9x - 7\right]$

Three cv from 1st Derivative (more from 2nd)

12. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval (0,10) ?

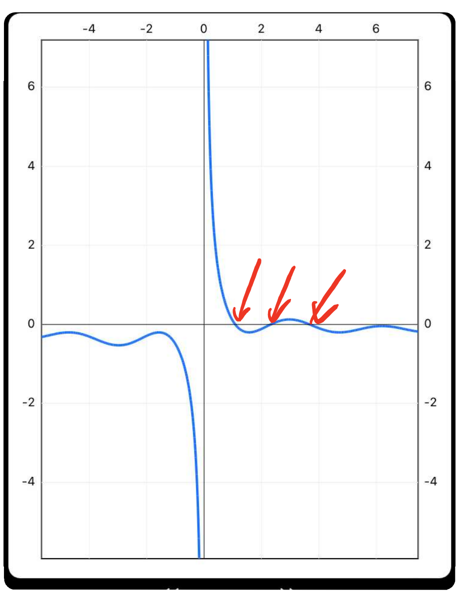
(A) One

(B) Three

(C) Four

(D) Five

(E) Seven



13. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$. X-int

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$. IVT

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$ Rolle's Theorem (Haven't learned)

(E) There is a number c in the open interval $(a\ b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. MVT Not Diff

14. The function f is defined for all x in the closed interval $[a, b]$ If f does not attain a maximum value on $[a, b]$, If f does not attain a maximum value on which of the following must be true? EVT does NOT apply

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.