Unit 5.3 Determining When F(x) is Increasing or Decreasing

APClassroom Hw 5.3 MC

1. Let f be the function defined by $f(x) = (\sin x)e^{-x}$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. On which of the following open intervals is f increasing? → $f' > 0$

(A) $\left(-\frac{\pi}{4}, \frac{\pi}{2}\right)$

(B) $\left(0, \frac{\pi}{2}\right)$ only

(C) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ only

(D) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

① $f' = \underbrace{\cos x \cdot e^{-x} + \sin x \cdot e^{-x}(-1)}_{\text{Product Rule}}$

$f' = \dfrac{\cos x - \sin x}{e^{x}}$

② $f' = 0$

$\cos x - \sin x = 0$

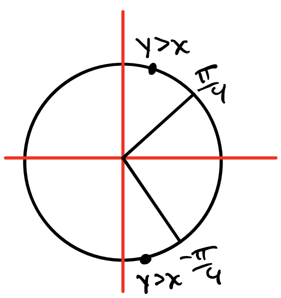
$\cos x = \sin x$

$x = \frac{\pi}{4}, -\frac{\pi}{4}$

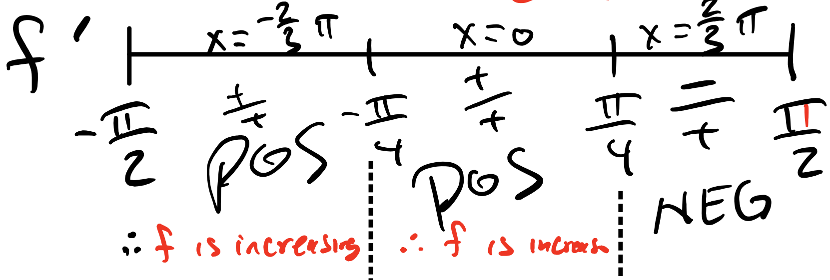
f' undefined

$e^{x} = 0$

no Solution



④ $f' = \dfrac{\cos x - \sin x}{e^{-x}}$ Always +



2. Let f be the function with derivative given by $f'(x) = x^2 - a^2 = (x-a)(x+a)$, where a is a positive constant. Which of the following statements is true?

(A) f is decreasing for $-a < x < a$ because $f'(x) < 0$ for $-a < x < a$.

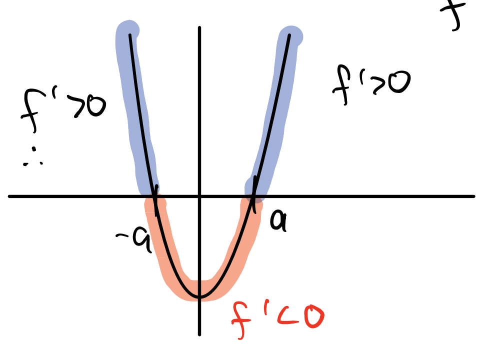
(B) f is decreasing for $x < -a$ and $x > a$ because $f'(x) < 0$ for $x < -a$ and $x > a$.

(C) f is decreasing for $x < 0$ because $f'(x) < 0$ for $x < 0$.

(D) f is decreasing for $x < 0$ because $f''(x) < 0$ for $x < 0$.

$f'(x) = x^2 - a^2$

parabola opens up



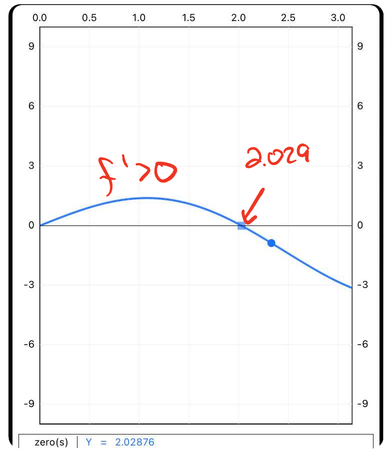
3. Let f be the function with derivative given by $f'(x) = \sin x + x\cos x$ for $0 \le x \le \pi$. On which of the following intervals is f increasing?

(A) $[0, 1.077]$ only

(B) $[0, 2.029]$

(C) $[1.077, \pi]$

(D) $[2.029, \pi]$ only



4. Let f be the function defined by $f(x) = (x + x^2)e^{-2x}$. On which of the following open intervals is f increasing?

(A) $\left(-\infty, \frac{-3-\sqrt{5}}{2}\right)$ and $\left(\frac{-3+\sqrt{5}}{2}, \infty\right)$

(B) $(-\infty, -1)$ and $(0, \infty)$

(C) $\left(-\infty, -\frac{\sqrt{2}}{2}\right)$ and $\left(\frac{\sqrt{2}}{2}, \infty\right)$

(D) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Product Rule

1 $f' = (1+2x)e^{-2x} + (x+x^2)e^{-2x}(-2)$

$f' = e^{-2x}\left(1+2x + (x+x^2)(-2)\right)$

$f' = \dfrac{-2x^2+1}{e^{2x}}$

2 $f'=0$

$0 = -2x^2+1$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

f' undefined

$e^{2x} = 0$

No Solution

$f' = \dfrac{1-2x^2}{e^{2x}}$ Always +

3 f': $x=-1$ | $-\sqrt{\frac{1}{2}}$ | $x=0$ | $\sqrt{\frac{1}{2}}$ | $x=1$

$\frac{-}{+}$ $\frac{+}{+}$ $\frac{-}{+}$

NEG POS NEG

$\therefore f$ is increasing

5. Let f be the function with derivative given by $f'(x) = x^2 - (a+b)x + ab = (x-a)(x-b)$, where a and b are constants such that $a < b$. Which of the following statements is true?

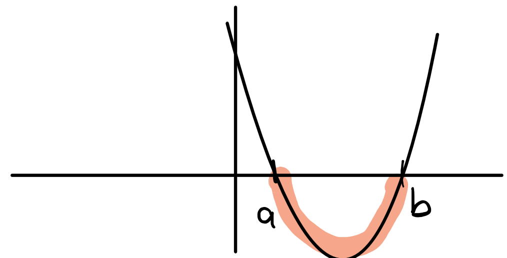
(A) f is decreasing for $a < x < b$ because $f'(x) < 0$ for $a < x < b$.

(B) f is decreasing for $x < a$ and $x > b$ because $f'(x) < 0$ for $x < a$ and $x > b$.

(C) f is decreasing for $x < \frac{a+b}{2}$ because $f'(x) < 0$ for $x < \frac{a+b}{2}$.

(D) f is decreasing for $x < \frac{a+b}{2}$ because $f''(x) < 0$ for $x < \frac{a+b}{2}$.

Parabola opens up



$f' < 0 \therefore f$ is dec.

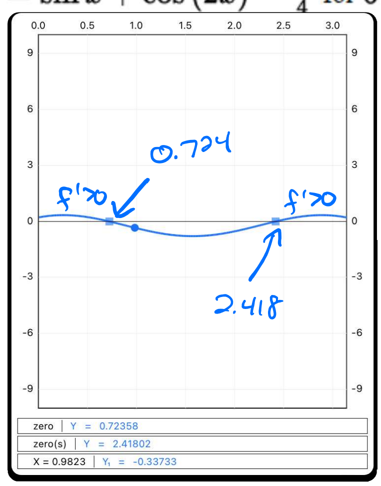
6. Let f be the function with derivative given by $f'(x) = \sin x + \cos(2x) - \frac{\pi}{4}$ for $0 \le x \le \pi$. On which of the following intervals is f increasing?

(A) $[0, 0.724]$ only

(B) $[0, 0.724]$ and $[2.418, 3.142]$

(C) $[0, 0.253]$ and $[1.571, 2.889]$

(D) $[0.724, 2.418]$



7. The first derivative of the function f is defined by $f'(x) = \sin(x^3 - x)$ for $0 \le x \le 2$. On what intervals is f increasing?

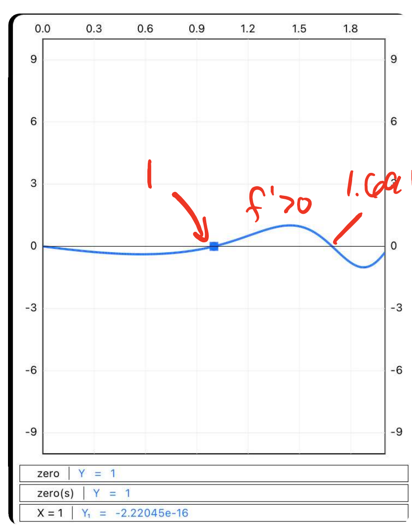
(A) $1 \le x \le 1.445$ only

(B) $1 \le x \le 1.691$

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$



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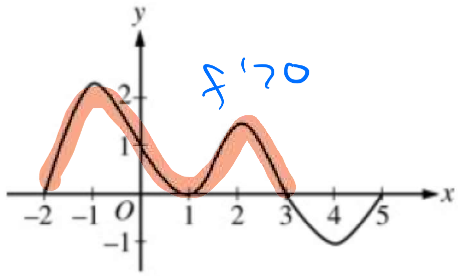


Graph of f'

The graph of f' the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2,1]$ only

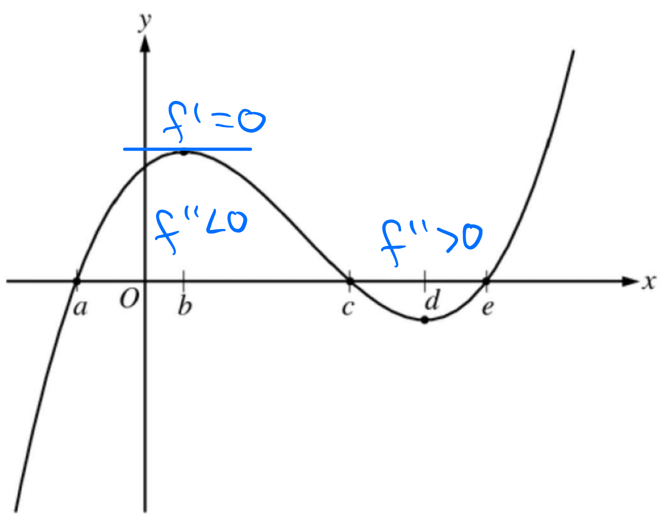
(B) $[-2,3]$

(C) $[3,5]$ only

(D) $[0,1.5]$ and $[3,5]$

(E) $[-2,-1]$,$[1,2]$ and $[4,5]$

9.



Graph of f

dec −

inc + above axis +

below axis −

The figure above shows the graph of the polynomial function f. For which value of x is it true that $f''(x) < f'(x) < f(x)$?

concavity up

+

concave down

−

(A) a

(B) b

(C) c

(D) d

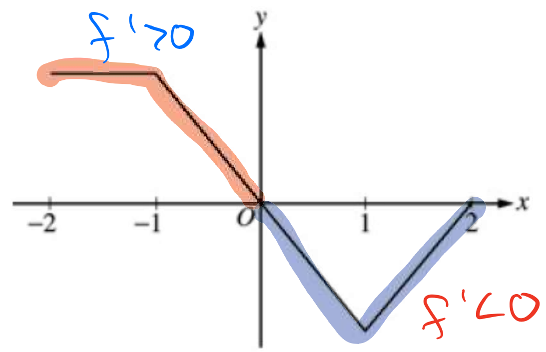
(E) e

$f(b) = +$

$f'(b) = 0$

$f''(b) = -$

10.



Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following statements is true about f?

(A) f is decreasing for $-1 \le x \le 1$.

(B) f is increasing for $-2 \le x \le 0$.

(C) f is increasing for $1 \le x \le 2$.

(D) f has a local minimum at $x = 0$.

(E) f is not differentiable at $x = -1$ and $x = 1$.

11. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$ Of the following, which is a possible value for $g(6)$?

(A) 15

(B) 18

(C) 21

(D) 24

(E) 27

x	g(x)
4	12
5	18
6	

+6

must be more than +6 b/c $g'' > 0$

any # > 24

12. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$
(B) $(-\infty, -7)$ and $(10, \infty)$
(C) $(-\infty, 10)$
(D) $(-10, 7)$
(E) $(-7, 10)$

① $g'(x) = x^2 + 3x - 70 = (x+10)(x-7)$

② $g' = 0$: $x = -10$, $x = 7$ | g' undefined: never

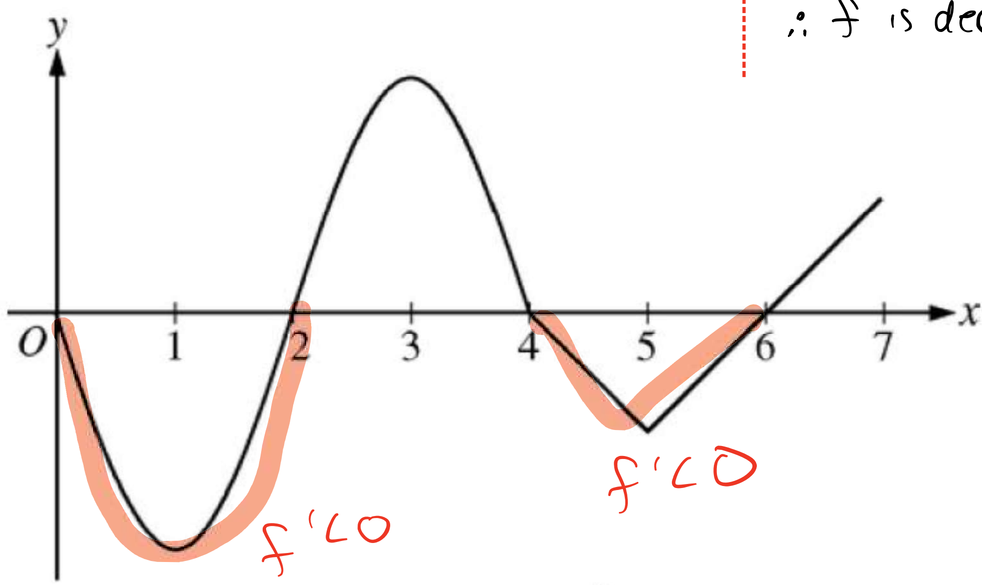
③ $g'(x) = (x+10)(x-7)$

f': $x = -100$ | $x = 0$ | $x = 100$

$(-)(-)$ -10 $(+)(-)$ 7 $(+)(+)$

POS | NEG $\therefore f$ is dec | POS

13.



Graph of f'

The graph of f', the derivative of the function f is shown above. On which of the following intervals is f decreasing?

(A) [2, 4] only
(B) [3, 5] only
(C) [0, 1] and [3, 5]
(D) [2, 4] and [6, 7]
(E) [0, 2] and [4, 6]

14. If $f'(x) > 0$ for all x, and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

Increasing Slowing down

(A)

x	$f(x)$
−1	4
0	3
1	1

dec

(C)

x	$f(x)$
−1	4
0	5
1	6

inc +1 +1 $f''=0$

(B)

x	$f(x)$
−1	4
0	4
1	4

constant

(D)

x	$f(x)$
−1	4
0	5
1	7

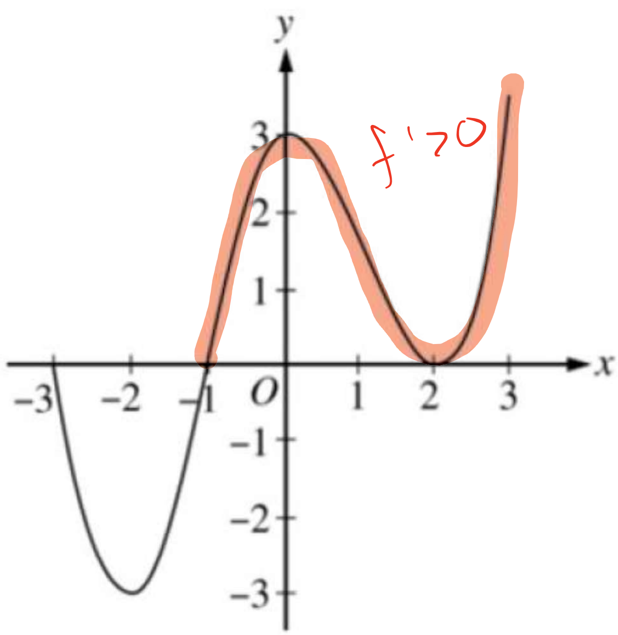
inc +1 +2 $f''>0$

(E)

x	$f(x)$
−1	4
0	6
1	7

inc +2 +1 $f''<0$

15.



Graph of f'

The graph of f', the derivative of the function f, is shown above for $-3 \le x \le 3$. On what intervals is f increasing?

(A) $[-3, -1]$ only

(B) $[-1, 3]$

(C) $[-2, 0]$ and $[2, 3]$

(D) $[-3, -1]$ and $[1, 3]$

16. The first derivative of the function g is given by $g'(x) = \cos(\pi x^2)$ for $-0.5 < x < 1.5$. On which of the following intervals is g decreasing?

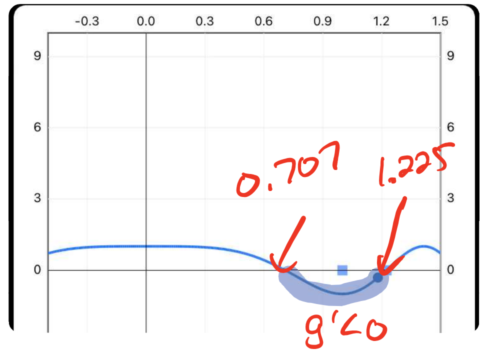
(A) $-0.5 < x < 0$

(B) $0 < x < 1$

(C) $0.707 < x < 1.225$

(D) $1.225 < x < 1.414$

(E) $1.414 < x < 1.5$



17.

x	−4	−3	−2	−1	0	1	2	3	4
$g'(x)$	2	3	0	−3	−2	−1	0	3	2

INC → DEC → inc →

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

(A) $-2 \le x \le 2$ only

(B) $-1 \le x \le 1$ only

(C) $x \ge -2$

(D) $x \ge 2$ only

(E) $x \le -2$ or $x \ge 2$

18. If $f(x) = x + \frac{1}{x}$, then the set of values for which f increases is (where is $f' > 0$?)

(A) $(-\infty, -1] \cup [1, \infty)$

(B) $[-1, 1]$

(C) $(-\infty, \infty)$

(D) $(0, \infty)$

(E) $(-\infty, 0) \cup (0, \infty)$

$\boxed{(x^{-1})' = -x^{-2}}$

$f' = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$

$f' = 0$	f' is undefined
$x^2 - 1 = 0$	$x^2 = 0$
$x^2 = 1$	$x = 0$
$x = \pm 1$	

$f'(x) = \frac{x^2-1}{x^2} = \frac{x^2-1}{\text{Always} +}$

f': $x=-10$ | -1 | $x = -\frac{1}{2}$ | 0 | $x = \frac{1}{2}$ | 1 | $x = 10$

$\frac{+}{+}$	$\frac{-}{+}$	$\frac{-}{+}$	$\frac{+}{+}$
POS	NEG	NEG	POS
f is inc	f is dec	f is dec	f is inc