Unit 5.4 1st Derivative Test

APClassroom Hw 5.4 MCQ

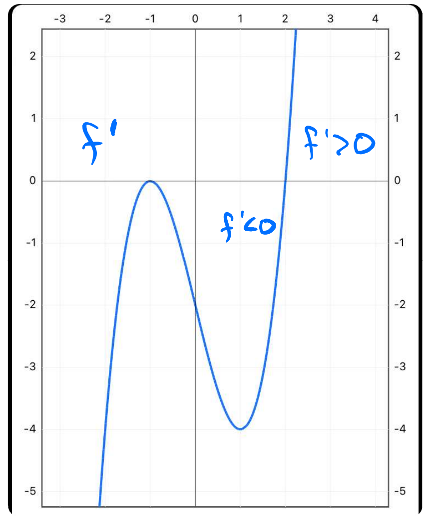
1. Let f be the function with derivative $f'(x) = x^3 - 3x - 2$. Which of the following statements is true?

(A) f has no relative minima and one relative maximum.

(B) f has one relative minimum and no relative maxima.

(C) f has one relative minimum and one relative maximum.

(D) f has two relative minima and one relative maximum.



- f' changes from negative to positive
  ∴ rel MIN
- f' never changes from positive to negative
  ∴ NO rel MAX

2. Let f be a differentiable function with a domain of $(0, 10)$. It is known that $f'(x)$, the derivative of $f(x)$, is negative on the intervals $(0, 2)$ and $(4, 6)$ and positive on the intervals $(2, 4)$ and $(6, 10)$. Which of the following statements is true?

(A) f has no relative minima and three relative maxima.

(B) f has one relative minimum and two relative maxima.

(C) f has two relative minima and one relative maximum.

(D) f has three relative minima and no relative maxima.

NEG POS NEG POS

0 2 4 6 10

Rel MIN @ x=2

Rel MAX @ x=4

Rel MIN @ x=6

3. The function f is defined by $f(x) = e^{-x}(x^2 + 2x)$. At what values of x does f have a relative maximum?

(A) $x = -2 + \sqrt{2}$ and $x = -2 - \sqrt{2}$

(B) $x = -\sqrt{2}$ only

(C) $x = -2$ and $x = 0$

(D) $x = \sqrt{2}$ only

$f' = e^{-x}(-1)(x^2+2x) + e^{-x}(2x+2)$

$f' = e^{-x}(-x^2-2x+2x+2)$

$f' = \dfrac{2-x^2}{e^x}$

$f' = 0$	f is undefined
$x = \pm\sqrt{2}$	NO Solution

$f' = \dfrac{2-x^2}{e^x} = \dfrac{2-x^2}{\text{Always}+}$

$x=-10$ | $x=0$ | $x=10$

NEG $-\sqrt{2}$ POS $\sqrt{2}$ NEG

MIN @ $x=-\sqrt{2}$

MAX @ $x=\sqrt{2}$

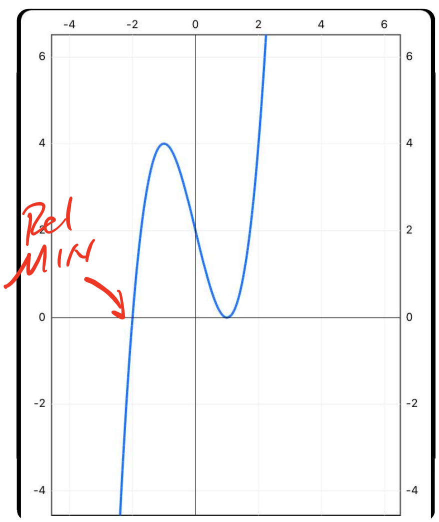
4. Let f be the function with derivative $f'(x) = x^3 - 3x + 2$. Which of the following statements is true?

(A) f has no relative minima and one relative maximum.

(B) f has one relative minimum and no relative maxima.

(C) f has one relative minimum and one relative maximum.

(D) f has two relative minima and one relative maximum.



- f' changes from negative to positive
  ∴ rel MIN
- f' never changes from positive to negative
  ∴ NO rel MAX

5. The function f is defined by $f(x) = x^2e^{-x^2}$. At what values of x does f have a relative maximum?

(A) -2
(B) 0
(C) 1 only
(D) -1 and 1

$f'(x) = 2x \cdot e^{-x^2} + x^2 e^{-x^2}(-2x)$

$f'(x) = 2xe^{-x^2}(1 - x^2)$

$f'(x) = \dfrac{2x(1-x^2)}{e^{x^2}}$

$f' = 0$	f' undefined
$x=0$, $x = \pm 1$	Never

$f' = \dfrac{2x(1-x^2)}{\text{Always}+}$

$x=-10$ | -1 | $x=-1/2$ | 0 | $x=1/2$ | 1 | $x=10$

POS, NEG, POS, NEG

Rel MIN @ $x=0$

Rel MAX @ $x=-1$

Rel MAX @ $x=1$

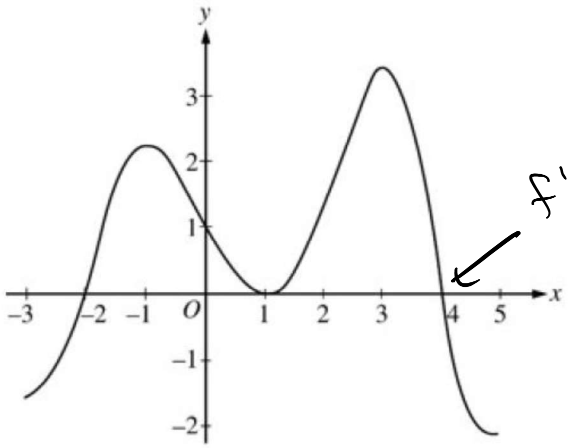
6. Let f be a differentiable function with a domain of $(0, 5)$. It is known that $f'(x)$, the derivative of $f(x)$, is negative on the intervals $(0, 1)$ and $(2, 3)$ and positive on the intervals $(1, 2)$ and $(3, 5)$. Which of the following statements is true?

(A) f has no relative minima and three relative maxima.
(B) f one relative minimum and two relative maxima.
(C) f has two relative minima and one relative maximum.
(D) f has three relative minima and no relative maxima.

0 | NEG | 1 | POS | 2 | NEG | 3 | POS | 5

Rel MIN (at 1), Rel MAX (at 2), Rel MIN (at 3)

7.



Graph of f'

f' changes from pos to neg
∴ f has rel MAX

The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) -2 only

(B) 1 only

(C) 4 only

(D) -1 and 3 only

(E) -2, 1, and 4

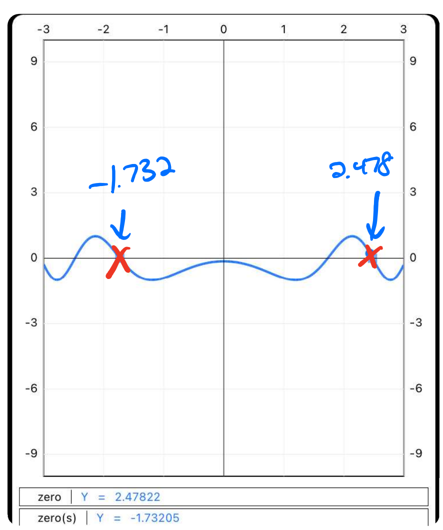
8. Let f be the function with derivative given by $f'(x) = \sin(x^2 - 3)$. At what values of x in the interval $-3 < x < 3$ does f have a relative maximum?

(A) -1.732 and 2.478 only

(B) -2.478 and 1.732 only

(C) -2.138, 0, and 2.138

(D) -2.478, -1.732, 1.732, and 2.478



9. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

(A) -3

(B) $-7/3$

(C) $-5/2$

(D) $7/3$

(E) $5/2$

$f' = 1(x-3)^2 + (x-2)\cdot \underbrace{2(x-3)'(1)}_{\text{Chain}}$

$f' = (x-3)[(x-3) + 2(x-2)]$

$f' = (x-3)(3x-7)$

$\underline{f'=0}$

$x=3$

$x=7/3$

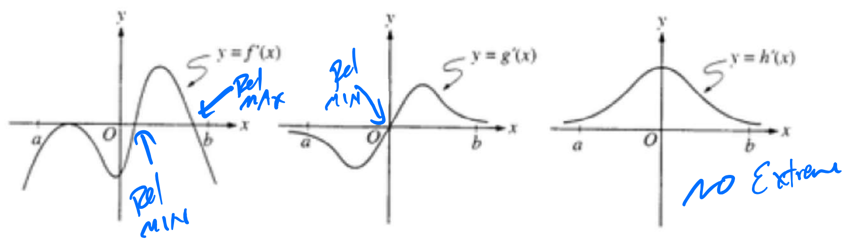
$\underline{f' \text{ undefined}}$

None

$f'=(x-3)(3x-7)$

$x=0$		$x=8/3$		$x=10$
POS	$7/3$	NEG	3	POS
	Rel MAX			

10.



The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a<x<b$?

(A) f only

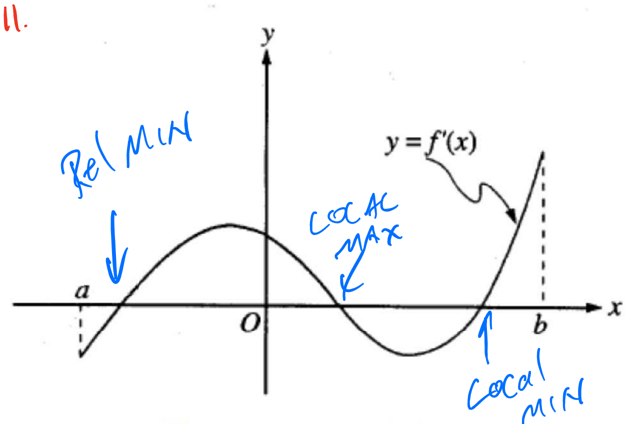
(B) g only

(C) h only

(D) f and g only

(E) f, g, and h

11.



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b) ?

(A) One relative maximum and two relative minima

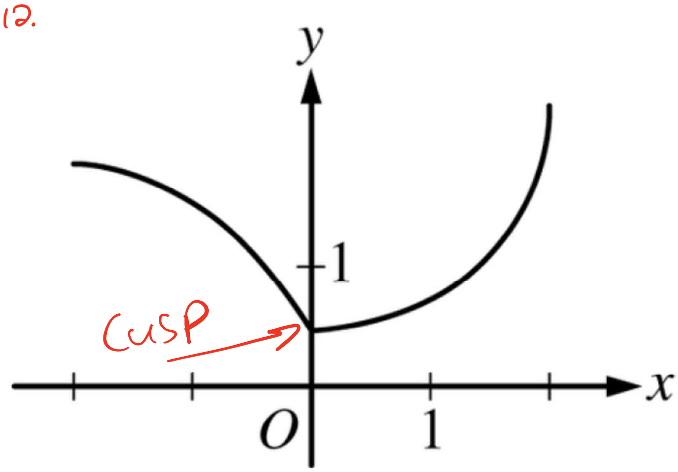
(B) Two relative maxima and one relative minimum

(C) Three relative maxima and one relative minimum

(D) One relative maximum and three relative minima

(E) Three relative maxima and two relative minima

12.



Graph of f

The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$.

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

13. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

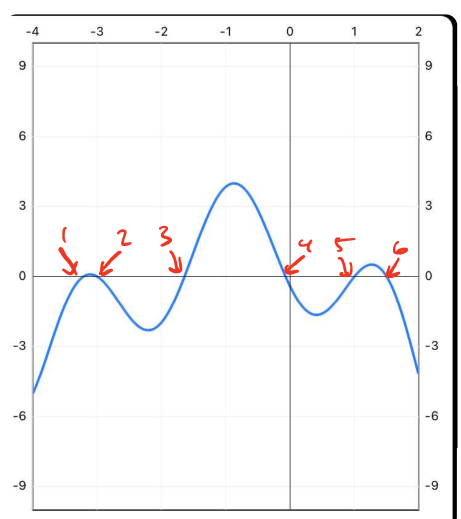
(A) Two

(B) Three

(C) Four

(D) Five

(E) Six

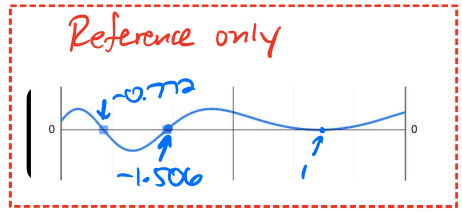


APClassroom Hw 5.4 FRQ

1. A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

Let f be a twice-differentiable function with $f(0) = 4$. The derivative of f is given by $f'(x) = \sin\left(x^2 - 2x + 1\right)$ for $-2 \le x \le 2$.

(a) Find all values of x in the interval $-2 < x < 2$ at which f has a critical point. Classify each as the location of a relative minimum, a relative maximum, or neither. Justify your answers.



- f' changes from positive to negative at $x = -0.772$
  $\therefore f$ has a relative min there
  +1
- f' changes from negative to positive at $x = -1.506$
  $\therefore f$ has a relative max there
  +1
- $f'(x) = 0$ at $x = 1$ but f' does not change sign there.
  $\therefore f$ does not have a relative min or max there.
  +1

(b) Use the line tangent to the graph of f at $x = 0$ to approximate $f(0.25)$.

PoT $(0,4)$

SoT $f'(0) = 0.841$

$y - 4 = 0.841(x - 0)$ +1

$f(0.25) \approx 4 + 0.841(0.25)$ +1

(c) On the interval $0 \le x \le 0.25$, $f'(x) > 0$ and $f''(x) < 0$. Is the approximation found in part (b) an overestimate or an underestimate for $f(0.25)$? Give a reason for your answer.

$f''(x) < 0$ at $x = 0.25$

$\therefore f$ is concave down there +1

$\therefore f(0.25)$ approximation is overestimate +1

(d) Using the Mean Value Theorem, explain why the average rate of change of f over the interval $-2 \le x \le 2$ cannot equal 1.25.

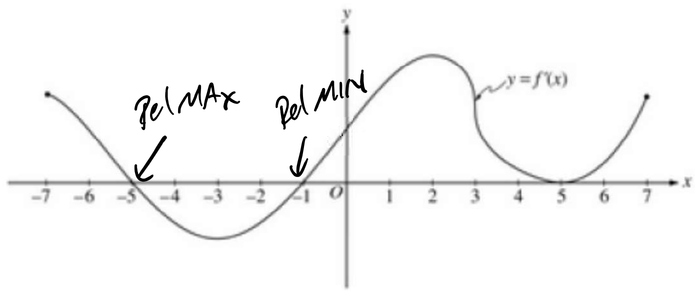
- f' is differentiable for $-2 \le x \le 2$
  $\therefore f'$ is continuous there.

+2

The MVT guarantees a c such that $f'(c) = \dfrac{f(-2) - f(2)}{-2 - 2}$

However, f' reaches a maximum of 1 $\therefore$ The ARC could never be 1.25

2.



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

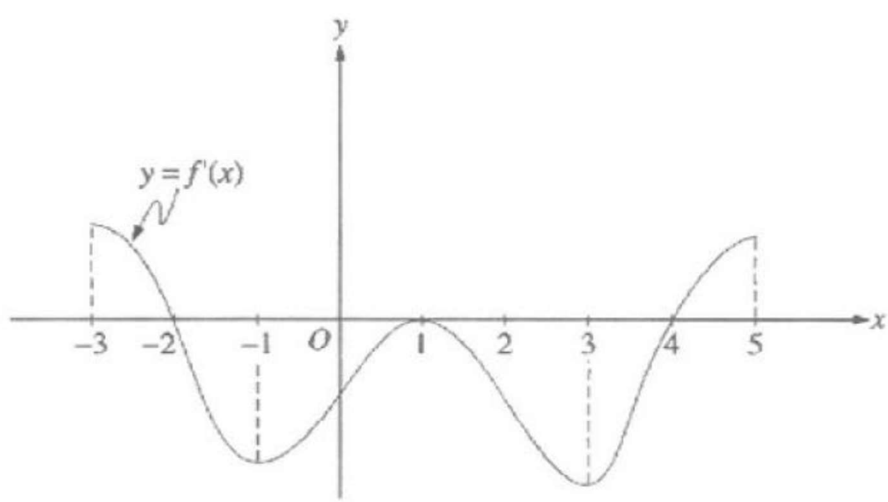
a. Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

f' changes from negative to positive at $x = -1$ +1
$\therefore$ f has a relative minimum there. +1

b. Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

f' changes from positive to negative at $x = -5$ +1
$\therefore$ f has a relative maximum there. +1

3.



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

a. For what values of x does f have a relative maximum? Why?

f' changes from positive to negative at $x = -2$ +1
$\therefore$ f has a relative maximum there. +1

b. For what values of x does f have a relative minimum? Why?

f' changes from negative to positive at $x = 4$ +1
$\therefore$ f has a relative minimum there. +1

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2-2}{x}$ for all $x \neq 0$.

$h' = 0$

Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

$h'(x) = 0$ at $x = -\sqrt{2}, \sqrt{2}$

$h' = \frac{x^2-2}{x}$

$x=-10$ | $x=-1$ | $x=1$ | $x=10$

NEG $-\sqrt{2}$ POS 0 NEG $\sqrt{2}$ POS

h' und

+1

- $h'(x)$ has horizontal tangents at $x=-\sqrt{2}$ and $x=\sqrt{2}$ +1
- $h'(x)$ changes from negative to positive at $x=-\sqrt{2}$ and $x=\sqrt{2}$ +2
  ∴ h has a relative minimums there.

5.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a maximum or a relative minimum at each of these values. Justify your answer.

f has a relative extremum at $x = 2$ +1

f' changes from positive to negative at $x = 2$
∴ f has a relative maximum there. +1